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THE PHYSICAL REVIEW

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THE

PHYSICAL REVIEW.

THE TONES FROM BELLS.

By Arthur Taber Jones.

Synopsis.

Normal Modes of Vibration.—The positions of the *nodal lines* are examined for partial tones up to the ninth. Certain partials are heard only when the bell is tapped on the sound bow, others only when it is tapped above the sound bow.

Elasticity.—A value is obtained for the *Young's modulus* of bell metal.

Striking Notes.—The *principal striking note* is the note which is best heard when tunes are played on bells, and which gives its name to a bell. It cannot be picked up from the bell by a resonator, it cannot be elicited from the bell by resonance, and it does not beat with a tuning fork of nearly its own pitch. No partial tone of the bell has the same pitch as this striking note, but the fifth partial is an octave above it. A striking note is not a difference tone, and it does not arise from compressional waves running through the material of the bell. The pitch of the striking note seems to be determined by the fifth partial, the octave in which it lies being, however, generally misjudged. A possible reason for this general failure in correct estimation of the octave is found in the rates at which the different partials reach their maximum intensities.

A *secondary striking note*, an octave below the fourth partial of the bell, can be heard when the bell is tapped above the sound bow.

1. Introduction.

BELLS have been a subject of interest to many investigators,[1] but from the physical point of view very little work has been done. Papers by Lord Rayleigh,[2] Canon A. B. Simpson,[3] and Mr. P. J. Blessing[4] contain practically all that is known.[5]

[1] For bibliography of more than 250 titles see H. B. Walters, Church Bells of England, London, Frowde, 1912.

[2] Phil. Mag., (5), 29, p. 1, 1890, or Theory of Sound, § 235*a*.

[3] Simpson's two popularly written articles on "Why Bells Sound Out of Tune" and "How to Cure Them" were published in the Pall Mall Magazine, Oct., 1895, and Sept., 1896. They have also been published in the form of a booklet, which I believe is out of print. London, Skeffington, 1897.

[4] Physikal. Zeitschr., 12, p. 597, 1911.

[5] From the last number of Science Abstracts (Jan., 1920) I learn of two publications on bells by J. Biehle. The copy of the Physikalische Zeitschrift in which these are reviewed (Vol. 20, pp. 429–431, Sept. 15, 1919) has not yet reached Smith College. I have, however, succeeded in seeing the review, and from it I judge that the material in my paper is sufficiently different from Biehle's to make it worth publishing. From the review of Biehle's work, from my study of the bells of the Dorothea Carlile chime, and from Simpson's statements about English and continental bells I think it likely that the bells of the Dorothea Carlile chime are more like English bells than like the 450 bells which Biehle has examined.

Rayleigh used a set of Helmholtz resonators and examined the partial tones given out by each of a number of bells. Each resonator was tuned by covering its mouth to a greater or less extent with a finger, and then the various partial tones of the bells were picked out by ear. In seven out of the eight church bells which Rayleigh examined the fifth partial was the only tone, irrespective of octave, which was close to the note that the bell was said to give, and in the case of at least one of the bells this fifth partial was an octave higher than the note which the founders gave as the pitch of the bell.

Simpson's description of his method of study is given in a single sentence: "Any note of a bell can be elicited separately by touching the bell with the stem of a vibrating fork which is of the same pitch as the note in question." Simpson examined a considerable number of bells and came to regard as the most important partials three tones which are usually in the neighborhood of three successive octaves. The lowest of these three is the *hum note*, which is the lowest tone given by the bell. The next is the so-called *fundamental*, and for the highest of the three Simpson introduced the term *nominal*. Simpson adduced reasons for believing that when English tuners tuned a peal of bells they tuned the nominals and paid little attention to the other tones, whereas continental tuners paid most attention to the fundamentals. He also remarks, "There is further this curious fact: That while a tuner [English] always gave the nominal as the note of the bell, he invariably gave the pitch an octave lower than it really was." Simpson pointed out that for the finest musical quality all the partial tones of a bell should harmonize. This they seldom do. The interval from the hum note for instance to the fundamental he found was usually less than an octave, and the interval from the fundamental to the nominal somewhat over an octave. Simpson suggested a method of tuning bells which appears to depend on the fact that different partials have nodal circles at different distances up the bell. Thus by thinning the bell at certain distances from the bottom certain partials may be more affected than others. The results thus obtained are said to be very fine.[1]

Blessing distinguished the *principal note* or *striking note* of a bell from its *secondary notes*. The striking note is the note which is usually most noticed when tunes are played on bells. The secondary notes are pro-

[1] Simpson's method of tuning was at once taken up and developed by John Taylor and Co. of Loughborough, Eng. As to the results see T. L. Papillon, Encycl. Brit., ed. 11, article Bell; Lord Grimthorpe, Clocks, Watches, and Bells, ed. 8, p. 393, London, Lockwood, 1903; W. W. Starmer, Carillons, p. 12, London, Novello, 1915; W. W. Starmer, Musical Times, 60, p. 522, Oct. 1, 1919. There are in North America three chimes by Messrs. Taylor—one of 10 bells at the Iowa State College, one of 13 bells at St. John's Church, Peterborough, Ontario, and one of 12 bells at the University of California—cast respectively in 1899, 1911, and 1915.

duced by division of the bell into segments separated by nodal lines, and are therefore the various partial tones of the bell. These partial tones can be picked out by properly tuned resonators and can be elicited from the bell by resonance. The striking note, on the contrary, sound no louder with a resonator than without it and cannot be elicited from the bell by resonance. The pitch of the striking note is usually not far from that of the second partial, but the manner in which the striking note is produced is a mystery.

These remarkable statements as to the striking note of a bell, and the curious fact observed by Rayleigh and Simpson that the pitch of the bell appears to be an octave below the fifth partial of the bell certainly suggest some interesting problems. The installing of the Dorothea Carlile chime of twelve bells[1] at Smith College provided opportunity for a study of the striking note and also for comparison of the partials of a number of bells, all cast in the same year and by the same founder.

2. Method of Study.

Pitch Determinations.—The partial tones were found by Rayleigh's method with the use of Helmholtz resonators. The striking notes were picked up without resonators. The pitches, both of partial tones and of striking notes, were obtained by ear by comparison with a sonometer which carried a wire one meter long. Two positions of the sonometer bridge were found such that a note from the sonometer was in one case just perceptibly sharper and in the other just perceptibly flatter than the tone in question, and the mean of these two settings of the bridge was used. In order to avoid errors due to large amplitude the sonometer wire was plucked gently and the tuning fork was tapped lightly. As standard a c_4 [= 512 vd] tuning fork by König was used, and the sonometer wire was frequently compared with it.

The accuracy of the pitch determinations seemed at the time of making an observation to be often as good as 5 cents,[2] but observations made at different times usually differed by much more than this. The average deviation of half a dozen to a dozen observations made at different times was usually not far from 15 cents to 20 cents.[3]

Calibration of Sonometer.—For the shorter lengths of the sonometer wire the frequency of the note from the wire was not accurately proportional to the reciprocal of the length. This appeared to be due

[1] Cast in 1919 by the Meneely Bell Company, Troy, N. Y. Total weight 11,838 lbs. [= 5370 kg.] Weights of largest and smallest bells respectively 3006 lbs. [= 1364 kg.] and 268 lbs. [= 122 kg.].

[2] In designating intervals I use Mr. Ellis's very convenient interval the *cent*, 100 of which make an equally tempered half step.

[3] 1 per cent. change in frequency corresponds to a change in pitch of 17.2 cents.

partly to the natural stiffness of the wire and partly to the increased tension when the bridge approached the end of the wire. In order to calibrate the sonometer two methods were used. For the range from c_3 to c_5 settings of the bridge were made at pitches which corresponded to a dozen selected notes on a reed organ which had a stop tuned to the untempered scale. Care was used to blow the organ at a pressure at which the tuning was good. For the range above c_5 it was at first thought that the first, second, third, etc., overtones of any part of the wire might be taken as the octave, twelfth, double octave, etc., of the note given by that part of the wire, and therefore that corresponding to any chosen position A of the bridge the positions for the octave, twelfth, double octave, etc., would be those which gave the same notes as the corresponding overtones obtained with the bridge at A. This, however, assumed that the natural stiffness of the wire was negligible and did not prove satisfactory. The method was therefore modified. No attempt was made to get the overtones of the wire, but the octave was estimated by ear, and only the octave was used. Thus corresponding to any chosen position A of the bridge two other positions were found, one at which the pitch sounded a trifle flatter and one at which it sounded a trifle sharper than the octave of the note given at A. The mean of these two was taken as the setting for the octave of the note given at A. In the range where this method overlapped the calibration from the reed organ the agreement of the two methods was very satisfactory. The corrections obtained have been applied to all readings, although for pitches below f_4 they are not more than 10 cents. In the neighborhood of c_5 they amount to about 18 cents, and in the neighborhood of g_5 to nearly 25 cents.

Temperature Correction.—The pitches if the bells appear to be very little affected by changes in temperature. Throughout the later part of the work the temperature was read at frequent intervals on a thermometer hung near the bells. The temperatures ranged from 10° C. to 30° C. The pitch of the hour bell—low f of the chime—in the range from − 15° C. to 0° C., and the pitches of both the transverse and the longitudinal vibrations of a straight bar of bell metal in the range from − 10° C. to + 20° C. were also determined. The temperature coefficients obtained in the different cases were small and were not consistent.

It is not difficult to show that for the transverse and longitudinal vibrations of a straight bar, and for the extensional and flexural vibrations of a thin ring,

$$n = n_0 \left(1 + \frac{\alpha + \gamma}{2} \cdot t\right), \qquad (1)$$

where n and n_0 stand for the frequencies when the bar is respectively at temperatures t and 0°, and α and γ stand for the temperature coefficients respectively of linear expansion and of Young's modulus. For many metals $\alpha + \gamma$ is negative, and the pitches of such bars and rings therefore fall with rise of temperature. Since equation (1) holds for all four of these cases it seems likely that it will also hold for bells. Taking α for bell metal as 0.000018 per degree C.,[1] and γ as $-$ 0.0003 per degree C.,[2] equation (1) leads to a temperature coefficient of about $-$ 0.24 cent per degree C.

In view of the lack of consistency in the experimental values for the temperature coefficient of pitch of the bells, and of the small value which such a correction would have, it was finally decided to make no attempt to reduce readings to a common temperature.

Pitch as a Function of Amplitude.—Throughout most of the work it was thought that the pitches of many of the partial tones—especially of the lower partials of the larger bells—were somewhat lower when the bells were first struck than when the sound had nearly died out. This effect was not confirmed by the frequency with which properly tuned forks would beat with the tones in question, and was probably a case of the subjective lowering of pitch which has been discussed by Dr. C. V. Burton.[3] The pitches given below are averages from values which were obtained when tapping the bells gently.

3. The Partial Tones of the Bells.

Nodal Lines.—Rayleigh determined the number of nodal meridians for the first five partials of two bells. His method was to find a number of successive meridians at which the beats of the tone in question vanished. The number of nodal meridians is then half the total number of the meridians at which the beats vanish. In the case of the sixth partial of the largest bell of the Dorothea Carlile chime this method failed. Instead of two normal modes of vibration which gave a single set of beats for this partial there seemed to be several normal modes of nearly the same frequency, so that there were beats of several different frequencies. The frequencies of the most prominent beats varied in an

[1] From the Landolt and Börnstein tables. This is about the value given for brass and for a bronze.

[2] From values found by Kiewiet [Winkelmann, Hbduch. d. Phys., ed. 2, Vol. 1, p. 567] for the temperature coefficients of various copper-tin alloys. Kiewiet's values have evidently been multiplied by some factor. On comparing his values with those for various substances as given in the Landolt and Börnstein tables I think it likely that this factor is 10^4, and have so assumed. The coefficient given above is for the proportion of Cu 78 per cent. to Sn 22 per cent, which is the proportion used by the Meneely Bell Company.

[3] E. H. Barton, A Text-book on Sound, p. 579.

erratic manner with the position at which the bell was tapped and the position at which the resonator was held, both along a circle of latitude and along a meridian. These variations may be partly due to the long inscription on this bell.

After some time spent in trying to unravel these different beats another method of determining the number of nodal meridians was hit upon which proved entirely satisfactory and was in most cases much more expeditious. The resonator was connected to the observer's head by a piece of rubber tubing and the binaurals of a stethoscope. A single position was found where the beats vanished, the bell was tapped at this one position, and the resonator was moved quickly some distance around the bell. At the nodal meridians the sound grew faint and between them swelled out. This method proved to be also of service in cases where no beats could be detected and Rayleigh's method would have involved fastening to the bell a local load.

As regards nodal circles, Rayleigh observed that certain partials were

TABLE I.

Nodal Lines.

In this table the positions of nodal circles are indicated by fractions. The unit chosen is the distance measured along the outer surface from the bottom of the bell to the point where the vertical "shoulder" joins the more or less horizontal "crown." Numbers in parentheses give the average deviations of the measurements on the different bells. The middle of the "sound bow" is at 0.164 (0.007).

Partial.	Bells on which Observed.	No. of Nodal Meridians.	Positions of Nodal Circles.	Remarks.
.1 = hum note	All	4	None	
2 = fundamental	All	4	0.33 (0.015)	
3	All	6	0.47 (0.012)	
4	All	6	None found	Not certainly detected when bell was tapped below 0.21 (0.014).
5 = nominal	All	8	0.48 (0.017)	Very clear when bell is tapped on sound bow. Faint for other positions of tapping. On five smallest bells not detected when tapping above sound bow.
6	6 largest	8	0.20 (0.009) 0.53 (0.033)	
7	6 largest	10	None found	Detected only when tapping below 0.37 (0.035).
8	2 largest	10	None found	Detected only when tapping above 0.46 (0.025).
9	1 largest	12	None found	Detected only when tapping below 0.37.

very faint, if heard at all, when the bell was tapped at certain latitudes. My results check his very well and extend the observations beyond the first five partials. They are given in Table I. It will be seen that vibration of the material in the sound bow—on which the clapper

TABLE II.

Pitches of the Tones.

In the upper half of this table pitches are given on the basis of $c_3 = 256$ vd. Calculations on the basis of $a_3 = 435$ vd. would indicate that all but one of the bells are more or less flat. A note with no sign following it means that the value obtained did not differ from that equally tempered note by more than 10 cents; a note followed by + or − means that the value obtained did not differ from that note by more than 35 cents; and two notes mean that the value obtained lay between them, but did not lie as close to either as 35 cents.

In the lower half of the table the pitches are in cents above the principal striking note. The three bottom lines give the average, the average deviation, and the range of values for the lower half of the table.

Bell.	Principal Striking Note.	Partial Tones.								
		1.	2.	3.	4.	5.	6.	7.	8.	9.
$e\flat$	$e\flat_3$	f_2+	$e\flat_3$	$g\flat_3 g_3$	$c_4 d\flat_4$	$e\flat_4+$	a_4	$b\flat_4+$	$e\flat_5+$	$e\flat_5 e_5$
f	f_3	$a\flat_2$	$e\flat_3$	$a\flat_3$	d_4+	f_4	$b\flat_4$	c_5+	$e\flat_5+$	
g	g_3	$a_2 b\flat_2$	$g\flat_3-$	$b\flat_3 b_3$	$e_4 f_4$	g_4+	$d\flat_5+$	d_5+		
$a\flat$	$a\flat_3$	b_2-	$g\flat_3$	b_3	$f_4 g\flat_4$	$a\flat_4+$	$d\flat_5$	$e\flat_5 e_5$		
a	a_3+	c_3	g_3	c_4	g_4-	$a_4 b\flat_4$	$e\flat_5-$	e_5+		
$b\flat$	$b\flat_3-$	$c_3 d\flat_3$	a_3	$d\flat_4+$	g_4	$b\flat_4$	e_5-	f_5		
c	c_4	$d_3 e\flat_3$	c_4+	$e\flat_4+$	$a_4 b\flat_4$	c_5+				
$d\flat$	$d\flat_4$	e_3-	b_3+	e_4-	$b\flat_4-$	$d\flat_5$				
d	d_4-	f_3	$d\flat_4+$	f_4	c_5-	d_5-				
$e\flat$	$e\flat_4+$	$g\flat_3$	$d\flat_4$	$g\flat_4$	$d\flat_5-$	$e\flat_5+$				
f	f_4+	$a\flat_3$	$e\flat_4+$	$a\flat_4$	$e\flat_5-$	f_5+				
g	g_4-	$a_3 b\flat_3$	$f_4 g\flat_4$	$b\flat_4+$	e_5	$g\flat_5 g_5$				
$e\flat$		−969	+ 2	+349	+ 948	+1235	+1813	+1934	+2414	+2466
f		−891	−203	+298	+ 930	+1200	+1693	+1923	+2213	
g		−936	−118	+341	+ 950	+1220	+1830	+1906		
$a\flat$		−939	−206	+292	+ 929	+1224	+1692	+1946		
a		−922	−220	+289	+ 964	+1239	+1768	+1910		
$b\flat$		−909	− 71	+358	+ 930	+1239	+1808	+1933		
c		−956	+ 9	+310	+ 943	+1210				
$d\flat$		−930	−182	+265	+ 872	+1197				
d		−875	− 65	+334	+1000	+1210				
$e\flat$		−915	−224	+283	+ 955	+1202				
f		−904	−188	+293	+ 961	+1204				
g		−926	−131	+333	+ 922	+1172				
Average		−923	−133	+312	+ 942	+1213	+1768	+1925	+2314	+2466
Av. Dev.		20	71	26	21	16	50	12	100	
Range		94	233	93	128	67	138	40	201	

strikes—has little to do with the production of partials 4, 6, and 8, so that these partials are relatively faint when the bell is struck in the

usual manner by a clapper, whereas in the production of partials 5, 7, and 9 the vibration of the material above the sound bow appears to be unimportant, and these partials—especially 5, which is the lowest of them —will be more strongly brought out by a blow of the clapper.

The Pitches of the Patrial Tones.—The pitches of the partial tones are given in Table II. It will be seen that it is only in a rather rough way that the successive partials form the same intervals for the different bells of the Dorothea Carlile chime. It will also be seen that in every case a partial of even order lies closer to the partial next above it than to the one next below, and an odd partial lies closer to the partial next below it than to the one next above. The averages for all bells are given in Table III.

TABLE III.

Average Interval from One Partial to the Next.

Partials	1, 2	2, 3	3, 4	4, 5	5, 6	6, 7	7, 8	8, 9
Intervals in cents	790	445	630	271	541	158	385	52

This difference in the intervals according as we pass up from an odd or an even partial may to some extent be understood by referring to Table I., where it is seen that the transition from an even partial to the next above it involves an increase in the number of nodal meridians, whereas the transition from an odd partial to the next above does not. For instance, on passing from the fourth partial to the fifth the width of each vibrating segment is reduced by about one fourth, whereas on passing from the fifth partial to the sixth the height of a vibrating segment is reduced by something like a half. If nothing else changed we should therefore expect a larger interval between the fifth and sixth partials than between the fourth and fifth. Table III. shows that this expectation is justified.

4. THE STRIKING NOTES.

Principal and Secondary Striking Notes.—From Table II. it will be seen that there is in general no partial which approximates at all closely to the pitch of the striking note. Thus Blessing's statement that a resonator does not respond to a striking note appears to be correct. Now when a bell is tapped on the waist or shoulder, *i.e.*, above the sound bow, it is known[1] that the bell sounds flatter than when tapped on the sound bow. And when tapping on the waist or shoulder I seemed to hear a note, flatter than the striking note, to which a resonator would not

[1] Helmholtz, Sensations of Tone, 4th Eng. ed., p. 72.

respond. I am therefore using the name *principal striking note* for the note obtained from the sound bow and not reinforced by a resonator, and the name *secondary striking note* for the note obtained from the waist or shoulder and not reinforced by a resonator. The secondary striking note is not as readily heard as the principal striking note. It is usually some 200 cents to 300 cents flatter than the principal striking note and somewhat flatter than the second partial of the bell.

Resonance.—On two of the bells of the Dorothea Carlile chime the principal striking note is very close to the second partial. At some half dozen different times I thought that I picked up with a resonator the principal striking note of some one of the other bells, but later attempts with the same bells seem to show that I was mistaken. A resonator does not, in general, respond to a striking note.

The results were similar when I attempted to get the bells to respond to a tuning fork which could be adjusted by moveable loads to give pitches throughout a range of about an octave. The bell was tapped and the fork adjusted by ear until its pitch was about that of the partial in question. The fork was then struck and its stem pressed against the bell. Various partial tones within the range of the fork responded clearly, but the striking notes did not thus respond.

Beats.—There was no difficulty in tuning a fork until its pitch was about that of a given partial and then hearing distinct beats when both fork and bell were tapped. But no beats could be detected when the fork had approximately the pitch of a striking note.

Difference Tones.—The first explanation of the striking notes which suggests itself is perhaps that they may be combination tones. Blessing says that Rudolph König suggested this possibility, but that no combination of the partial tones of a bell would give the proper frequency. Blessing, however, gives no data in support of his statement, and it seemed worth while to examine the question.

If a striking note is a combination tone of any sort it is most likely that it is a first order difference tone. From the observed frequencies of the various partials the difference tones given in Table IV. have been calculated. This table has reference to the principal striking notes. A calculation for the secondary striking notes shows similar results.

On comparing with Table II. it will be seen that Table IV. includes the difference tones arising from the fifth and seventh partials for all the bells on which the seventh partial was observed. But it will also be seen that in only half of these cases does the difference tone which arises from the fifth and seventh partials lie within a quarter of a step from the principal striking note. Moreover if the striking note were a combina-

TABLE IV.

Difference Tones.

This table includes all the first order difference tones which lie within half a step from the principal striking note. Those which lie within quarter of a step are starred.

Partials.	Bell.	Cents from Prin. Strik. Note Up to Dif. Tone.
2,5	Low *e♭*	+58
	c	+13*
	d	+85
	High *g*	+75
4,6	Low *f*	−91
	a♭	−79
	a	+59
5,7	Low *e♭*	+28*
	Low *f*	+71
	Low *g*	−22*
	a♭	+79
	a	−55
	b♭	+15*
7,8	Low *e♭*	−40*

tion tone we should expect it to be very faint, if heard at all, when the bell is tapped gently. As a matter of fact, the principal striking note comes out clearly when the bell is tapped gently. It seems then to be clear that a striking note is not a difference tone.

Compressional Waves.—It seemed possible that the striking notes might be due to compressional waves which spread through the material of a bell and returned periodically to the point where the bell had been struck. If this were the case the frequency of the principal striking note would be roughly the same as that of the longitudinal vibration of a straight bar which had a length equal to half the circumference of the sound bow.

To enable me to examine this matter the Meneely Bell Company kindly cast for me a rod of bell metal. After the ends had been trued off this rod had a length of 93.0 cm. The frequency of longitudinal vibration of the lowest mode was about 1810 vd.[1] To have the same frequency as that of the low *e♭* bell (304 vd) a rod of bell metal would therefore have to be 553 cm. long. The diameter of the mouth of the

[1] The density of this rod is about 8.86 g./c.c. Its Young's modulus is therefore about $10.0 \cdot 10^{11}$ dynes/cm^2. This value for the Young's modulus [bell metal = 78 per cent. Cu and 22 per cent. Sn] fits excellently on a curve coördinating the values obtained by Voigt [Wied. An., 48, p. 674, 1893] for copper, tin, and an alloy of 88 per cent. Cu and 12 per cent. Sn. Voigt's values were obtained by forcing a short bar of the given material to vibrate with a frequency of from one to two vibrations per second.

low *e♭* bell is 134 cm., so that the rod required would have a length much greater than half the circumference of the sound bow—in fact considerably in excess of the entire circumference of the mouth. Similar statements hold for the other bells.

Moreover, if the principal striking note were due to a compressional wave running, say, around through the sound bow, the diameters of the sound bows of the various bells should be inversely proportional to the frequencies of the principal striking notes. A calculation of the relative frequencies of the bells on this basis leads to values which are considerably in error—in the cases of three of the bells by something like 100 cents.

From this failure of the diameters of the bells to be proportional to the periods of the principal striking notes, and from the fact that a compressional wave would run through the material of a bell too quickly to give the observed pitch, it is clear that the principal striking note is not produced by a compressional wave.

Misjudged Octave.—From Table II. it will be seen that the pitch of the principal striking note is not far from an octave below the fifth partial of the bell. This checks with Rayleigh's and Simpson's observations that in most cases the only partial which is close to the pitch of a bell is the fifth, and that this fifth partial is an octave higher than the pitch of the bell is supposed to be.

It is well known that an error of an octave in judging the pitch of a note is easily made, and Simpson evidently believed that there is in general no note which has the pitch of the principal striking note, but that we hear most clearly the fifth partial of the bell and think it is an octave lower than it is. If that is the case it would explain why a striking note cannot be picked up from a bell by a resonator nor elicited from the bell by resonance, and why it will not beat with a tuning fork. It would probably also explain a statement made by Blessing that if the sound bow of a bell is gradually turned thinner and thinner the striking note grows fainter and fainter and finally disappears. Blessing says nothing as to how this process affects the partial tones of the bell. But since the fifth partial appears to be produced almost entirely by vibration of material in the sound bow it seems likely that reducing the thickness of the sound bow would weaken the fifth partial.

What evidence is there as to the octave in which the principal striking note lies? There is the judgment of the founders and tuners referred to by Rayleigh and Simpson. As to myself, I have at times felt very sure that the note I heard was really of the pitch which I have called that of the principal striking note and was not an octave higher. At other times

I did not feel so sure. When the chime was being played I have compared the notes which I heard with those on a reed organ. When doing this it seemed to me there could be no question but that the pitches really are those which they are ordinarily supposed to be, *i.e.*, each one an octave below the fifth partial. A statement from the Meneely Bell Company also corroborates this. With reference to the low *f* bell of the Dorothea Carlile chime they write me, "This is supposed to be about F natural above middle C, although bells as a rule, from their nature, seem to sound lower." The evidence seems to show that if the pitch of the principal striking note is determined by the fifth partial the octave is very generally misjudged—even by bell founders.

Is there any probable reason for such an error in judgment? In attempting to answer this question we may exclude from consideration the fourth, sixth, and eighth partials, all of which are faint, if heard at all, when the bell is struck on the sound bow. From Table II. it will be seen that the seventh partial is almost exactly a musical fifth (700 cents) above the fifth partial. Now in the harmonic series of tones to which the notes from strings, pipes, etc., approximate, we are accustomed to hear a fundamental accompanied by its octave and twelfth, so that even if the fifth and seventh partials do not produce a combination tone an octave below the fifth partial, they may nevertheless help to suggest a fundamental note of that pitch.

Another, and probably much more important, reason for misjudging the octave lies in the rates at which the different partials reach their full intensities. The fifth partial seems to reach its maximum intensity almost as soon as the bell is struck, the second and third, especially on the larger bells, not quite so soon. Thus it is possible that when a bell is struck the fifth partial at once attracts attention, and the second and third add a considerable volume of sound so soon afterward as to make the pitch seem an octave lower than that of the fifth partial. On the smaller bells the second and third partials seem to be more prompt in their response, and this may have something to do with the difficulty which bell founders are said to have usually experienced in casting small church bells of good musical quality.

My present hypothesis as to the principal striking note of a bell is, then, that it is a note of which the pitch, except for octave, is determined by the fifth partial, and that the octave in which we think we hear it is determined by the more sluggishly responding second and third partials. It is desirable that a considerable number of bells should be investigated, and that photographic records should be obtained showing the rates at which the various partials, especially the second, third, and fifth,

grow to their maximum intensity when the bells are struck in the usual manner by a clapper.

In conclusion I wish to express my thanks to the Meneely Bell Company for their kindness in giving me various data and in casting for me the rod of bell metal.

Smith College,
March 30, 1920.

THE VALENCY OF PHOTO-ELECTRONS AND THE PHOTO-ELECTRIC PROPERTIES OF SOME INSULATORS.

BY M. J. KELLY.

SYNOPSIS.

Photo-emission from insulators; the valency of photo-electrons.—Charged droplets of different insulators are suspended in the field of a parallel plate condenser by regulating the strength and direction of the electric field so that the electric force on the droplet is equal and opposite the gravitational force. Ultra-violet light of various intensities and frequencies is allowed to fall on the droplets and the photo-emission is observed. The photo-emission of several insulators is studied in this manner. By making the light intensities sufficiently low, the author finds that *only one electron escapes at each emission.* This is true for all insulators studied and is verified in the case of each insulator by the observation of several hundred emissions. This result is in line with similar work on ionization of gases by X-rays, γ-rays, β-particles, and α-particles by Professor Millikan and his students. By the use of different absorption screens *the long wave-length limit of photo-emission* from sulphur, shellac, oil and paraffine is located. These are located with greater accuracy than given by any other methods. The effect of water vapor and various surface impurities on photo-emission is discussed.

I. INTRODUCTION.

IN 1911 Millikan and Fletcher[1] furnished the first conclusive evidence that the mechanism of ionization by X-rays, gamma rays and β particles consisted in the detachment from the neutral molecule of one single elementary charge. Millikan, Gottschalk and Kelly[2] by using a similar method found the same to be true for ionization by α particles of several different gases and vapors. Mr. J. B. Dereux,[3] at Professor Millikan's suggestion, using the same general method studied the photo-emission from mercury drops. This work gave evidence which indicated that only one electron was expelled from the mercury atom in the process of photo-emission.

It would indeed be interesting if high speed particles of atomic size, high speed electrons and electromagnetic radiations of as widely differing frequencies as that of gamma rays and ultra-violet light all showed the uniform property of liberating one and only one electron from the atom in its ionization.

[1] Millikan and Fletcher, Phil. Mag. (6), 21, p. 753, 1911.

[2] Millikan, Gottschalk and Kelly, PHYS. REV.

[3] J. B. Dereux, PHYS. REV., 11, p. 276, 1918.

As mentioned above Mr. J. B. Dereux suspended single mercury drops, charged statically in the process of vaporization in the electric field of a parallel plate condenser and allowed ultra-violet light to fall directly on the drops. At the instant of a change in speed, indicating a change in the charge on the drop by its photo-emission, the light was shut off and its speed measured. It was possible to calculate from the change in speed the number of electrons liberated. The majority of the changes observed were those corresponding to emission of single electrons. Since a number of changes corresponded to emissions of more than one electron it was thought worth while to extend the investigation to other substances.

If the changes observed which correspond to emission of more than one electron were due to simultaneous emissions from different atoms on the surface, the chance of this could be decreased by using less active surfaces and decreasing the intensity of the radiation. In choosing surfaces whose emission currents would be small the insulators would naturally suggest themselves. They were selected for the further reason that this is the only method of study of photo-emission which is capable of furnishing very direct and very reliable information with regard to the photo-electric properties of insulators.

II. HISTORICAL SURVEY OF PHOTO-ELECTRIC WORK ON INSULATORS.

In the study of photo-emissions of insulators the general method of procedure has been to place a thin sheet of the insulator over the surface of one of the plates of a condenser, to apply an accelerating field, allow the radiation to fall on the surface and measure the current. In good insulators the surface charges up positively as the photo-electrons escape, until the potential gradient at the surface is sufficiently high to neutralize the accelerating potential and prevent the escape of further electrons. This gives a fatigue effect which makes consistent results difficult to obtain. Sheets of good insulators will remain for a long time in this polarized state and fresh samples of the material must be used. Using this general method Goldmann and Kalandyk[1] investigated the photo-electric effect in sulphur. Photo-electric currents were obtained when a source of ultra-violet light was used and the polarization effect mentioned above was encountered and studied. They found that the photo currents completely disappeared when a plate of glass was interposed between the source and the sulphur, which would indicate the long wave limit somewhere below λ 3200. R. Reiger[2] investigated a number of insulating

[1] Ann. d. Phys., XXXVI., p. 589, 1911.

[2] R. Reiger, Ann. d. Physik., XVII., p. 935, 1905.

materials, and found a number including, ebonite, mica, sealing wax and glass that showed small emission currents when exposed to the light of an electric arc. W. Wilson[1] examined shellac and found no emission with ultra-violet light.

III. EXPERIMENTAL.

The apparatus as described by Millikan[2] was used in the present work, with such changes as were found necessary in the problem. The apparatus consisted essentially of a parallel plate condenser with plates horizontal, the plates were 22 cm. in diameter and were separated 1.586 cm. An ebonite strip surrounded the condenser, with windows set in at proper intervals; one for the illumination of the drop, one for observation and one of quartz for admission of the ultra-violet light. Several small holes were bored in the center of the upper plate for the admission of drops and an electromagnetically controlled shutter was provided to close these holes when the drops were under observation. The condenser plates were placed in an air tight iron tank about 30 cm. in diameter and 50 cm. high. This cylinder was placed in a tank, about 50 cm. in diameter leaving a space between the walls of tank and cylinder which was filled with oil to hold the temperature constant. Tubes at the level of the condenser provided with vacuum tight, transparent windows ran from the outer tank into the inner tank at the proper places for admission of light for illuminating the drop, for observation, and for admitting the ultra-violet light. The windows in the ebonite strip were directly in front of these tubes.

The space between the condenser plates was illuminated by the light from a right-angled carbon arc. The light was focused on the drop by a cylindrical lens. A telescope having a high magnifying power was used for observing the drops. In the focal plane of the eye piece was a scale, the smallest division of which corresponded to 2 mm. Observations were made through a window 150 degrees from the one through which the light entered. A stop watch was used in timing.

The potential of the plates was furnished by 3,500 small lead cells giving a potential of about 7,000 volts. By means of a special switch the condenser could be charged, short circuited, or reversed by changing the position of a handle.

For changing the charge on the drops X-rays were used. The X-ray tube was placed just outside the apparatus near one of the windows. The source of radiation, usually a mercury vapor lamp was placed on a line of centers with the center of the condenser, and opposite a quartz window.

[1] W. Wilson, Ann. d. Physik., XXIII., p. 127, 1907.

[2] R. A. Millikan, PHYS. REV., 32, p. 349, 1911.

The drops of the various insulators were formed by getting the insulator in the liquid state and discharging it through an atomizer placed in the upper part of the iron cylinder and manipulated by air pressure from without. After atomizing there was always a rain of drops through the holes of the upper plate and one of proper gravity speed was selected; the shutter over the holes closed and the drop balanced by regulating its charge with X-rays. An electromagnetically controlled shutter over the quartz window was then lifted and the radiation allowed to enter until a change in speed showed an emission to have occurred, the shutter was then closed and the speed determined. From this speed and its speed under gravity the number of unit charges on the drop could be obtained and the difference between the number before and after emission gives the number of electrons emitted.

The number of charges on the droplet was determined from the relation

$$n = C\left(1 \pm \frac{t_1}{t_2}\right)\frac{1}{V},^1$$

where n = number of unit charges.

C = a constant for a given drop.

t_1 = number seconds it takes drop to traverse a certain distance d under gravity.

t_2 = number seconds it takes drop to traverse the same distance d under the field V.

V = potential across plates.

In determining the spectral range of photo-sensitiveness of the various insulators the method followed was to balance a drop in the field and allow radiations of a certain range to fall on it and observe whether or not emissions occurred. A large number of drops were used for each range so that there was no uncertainty in any case.

IV. SULPHUR.

Drops of sulphur were obtained by heating chemically pure sulphur to a proper temperature in a specially made atomizer. The atomizer was placed in the large chamber about 20 cm. above the upper plate of the condenser and heated by a coil which kept the entire atomizer at a temperature of 150° C. At this temperature the sulphur is in the form of a pale yellow mobile liquid and atomizes quite readily. The temperature regulation had to be close in order to obtain satisfactory performance for at about 160° C., the sulphur changes to a dark viscous liquid which is too viscous to permit being atomized. The sulphur drops were always charged statically and a satisfactory drop was easily obtained.

[1] Millikan and Fletcher, Phil. Mag. (6), 21, 753, 1911.

At the beginning of the work no special precautions were taken with the gas content of the chamber; it consisted of the air of the room freed fairly well of water vapor. The sulphur drops evaporated very rapidly; in fact so fast that all calculation of the number of charges on the drop or the changes in charge were made impossible by the enormous change in the size of the drop. Table I. gives the gravity speeds, taken at 5-minute intervals, of a drop held under observation for 30 minutes. The charge on the drop was varied by X-ray ionization so that as it evaporated it remained just balanced in the field. At 5-minute intervals the electric field was removed and readings were taken of its speed under gravity.

TABLE I.

Sulphur Drop No. 8.

Time to pass over 50 divisions:

1.	8.2 seconds.		
2.	10.4 "	at the end of	5 minutes.
3.	13.6 "	10	"
4.	17.7 "	15	"
5.	28.0 "	30	"

While evaporating the drops drifted badly and were soon out of the illuminated space. It was necessary to get rid of the evaporation before observations could be made. After a number of unsuccessful attempts the trouble was eliminated by boiling sulphur in the sealed chamber until the walls were coated with flowers of sulphur, and moisture-free air then allowed to stand in the chamber for days before the observations were taken. While the drops still evaporated, the rate was slow enough so that no uncertainty was introduced into the calculations. Table II. gives data on a drop observed for one hour in the same manner as the drop in Table I.

The readings given in Table II. for each time interval are the average of a number taken at each of the intervals. I was considerably surprised to find a variation of as much as 10 per cent. in successive timings, one

TABLE II.

Drop No. 27.

Time to pass over 50 divisions:

1.	7.6 seconds.		
2.	7.6 "	at end of	15 minutes.
3.	7.4 "	30	"
4.	7.0 "	60	"

taken immediately after the other as from experience in timing drops in another investigation I had acquired sufficient skill to check readings to .1 seconds. Upon examining a number of the drops under a micro-

scope they proved not to be perfect spheres, but spheres with minute crystal faces on the surface. The amount of this deformation varied considerably from drop to drop being apparently absent in some. This seemed to offer grounds for a satisfactory explanation of the variations, for a change in the orientation of the drop between two different excursions would change the resistance offered to its fall. In selecting drops for observation several timings of their gravity speed were taken one immediately after the other and if the variations were large the drop was discarded and another obtained.

In studying the valency of photo-electrons the following procedure was employed; after a drop with a proper gravity speed had been selected it was given a sufficient positive charge by X-ray ionization to just balance it in the field with the voltage used. The shutter covering the quartz window was opened and the radiations from a quartz mercury lamp allowed to fall on the drop until an increase in speed indicated an emission. The shutter was immediately closed and the speed determined. The shutter was again opened and the process repeated. This was continued until 4 or 5 emissions had occurred, then the drop was brought back to its initial number of charges with X-rays and the process repeated. Some of the smaller drops were started with an excess of 3 or 4 electrons and emissions allowed to occur until the drop had lost 3 or 4 electrons beyond the neutral point. This was not possible with the larger drops due to the electric force with the small number of unit charges and with voltages available being insufficient to balance the force of gravity. Some drops were kept under observation for two hours. 57 separate emissions were observed on one drop and over 500 emissions were observed under good experimental conditions and calculated.

When the observations were first begun the drops captured about as many electrons as they emitted. This was at first attributed to residual radio-activity as this same apparatus had been used by Millikan, Gottschalk and Kelly (loc. cit.) in studying ionization by α particles and had had radium open in the chamber. But on running a blank, that is observing the drop for an hour with the shutter closed, the drop was found to capture but two electrons; so it was concluded that the plates were emitting. In order to prevent this the plates were covered with lamp black, but the photo-sensitiveness was increased several hundredfold, in fact, as soon as the shutter was opened the drop would capture electrons in such rapid succession that the individual captures were indistinguishable. This emissivity was most probably due to the carbon in the lamp black as it was found that its long wave limit was λ 2550 (approx.) and this is about the limit Hughes[1] found for carbon.

[1] Hughes, Photo-Electricity, p. 102.

Shellac was then tried, as W. Wilson (loc. cit.) had pronounced it not photo-sensitive to the light of a quartz mercury-vapor lamp. However, it emitted electrons freely, so a thin coating of paraffine was next tried and no emission was found.

Table III. gives the results of observation on 10 droplets of sulphur.

TABLE III.

Drop No.	No. Minutes Under Obs.	No. Emissions.	No. Singles.	No. Mult.
21	90	35	35	
22	100	40	40	
24	100	44	44	
25	120	57	56	1
26	30	12	12	
28	60	38	37	1
29	110	45	45	
30	120	40	40	
31	60	20	20	
32	100	25	25	

As is shown all emissions with the exception of two were of a single electron and both of these were emissions of two electrons. As is evident from the " times under observation " the emissions were very frequent, in fact more so than the table indicates as this time includes time consumed in regulating charge on the drop, timing it and adjusting the illumination.

In order to decrease the rate of emissions a plate of fused quartz about 5 mm. thick was placed in the path of the beam. From evidence to be discussed later this plate appeared to cut off all radiation below $\lambda 2250$. This decreased the emission rate to six to eight an hour. About 200 emissions were observed at this reduced rate and 40 hours were consumed in observation. In every instance the emissions were unmistakably emissions of single electrons. Table IV. gives the data on a representative drop.

The activity of various drops differed considerably. The rate of emission from some drops was so slow that after observing them for a time to be sure they possessed activity they were discarded. This difference in activity must be attributed to surface conditions. All drops were most active initially and their activity gradually decreased with time; the rate of emission was decreased roughly one half at the end of two hours.

The surface polarization effect which was observed in sheets of sulphur was somewhat evident with the drops. If electrons were emitted until the drop became positively charged then as the positive charge increased

TABLE IV.

Drop No. 32.

t_1.	t_2.	K. V.	$\left(1 \pm \frac{t_1}{t_2}\right)\frac{1}{KV}$.	No. Units on Drop.	No. Electrons Emitted.
4.1	80.0	24	.0438	4 Pos.	
	12.6	24	.0552	5	1
	7.0	24	.0660	6	1
	4.9	24	.0765	7	1
4.1	19.5*	36	.0219	2 Pos.	
	22.4	36	.0330	3	1
	7.1	36	.0436	4	1
	12.2	24	.0557	5	1
	6.8	24	.0672	6	1
	5.0	24	.0756	7	1
4.0	5.0	24	.0750	7 Neg.	
	6.9	24	.0658	6	1
	12.8	24	.0550	5	1
	80.8	24	.0436	4	1
	19.0*	24	.0328	3	1
4.0	21.6	36	.0326	3 Pos.	
	7.0	36	.0434	4	1
	12.0	24	.0552	5	1
	6.8	24	.0660	6	1
	4.8	24	.0763	7	1
3.9	6.7	24	.0659	6 Neg.	
	12.0	24	.0551	5	1
	78.0	24	.0435	4	1
	19.8*	24	.0334	3	1
3.8	20.0*	36	.0221	2 Pos.	
	22.0	36	.0325	3	1
	6.9	36	.0430	4	1
	12.0	24	.0547	5	1
	6.6	24	.0656	6	1
	4.9	24	.0738	7	1

All t_2 marked with * the drop was going downward due to the force of gravity being greater than the opposite directed force of the electric field. In these cases the minus sign is used in the formula $\left(1 \pm \frac{t_1}{t_2}\right)$.

by further emissions, the rate of emission was greatly decreased. A drop that had lost 25 electrons beyond its neutral value was held under observation for one hour and only three emissions observed. The excess positive charge was neutralized by X-ray ionization and the drop was found to emit at the rate that was to be expected after it had been under

observation for one hour. Data was taken on a number of drops with varying excess of positive charge hoping to find some quantitative relation of stopping potentials but the variation from drop to drop, probably due to differences in surface contamination masked any such relation, if it exists.

In order to find the long-wave limit of emission for the sulphur a Hilger quartz spectroscope was set up outside the quartz window and by means of extra quartz lenses the radiation from any desired frequency was focused at the center of the condenser in the path of the drops. No emissions were obtainable with any frequency. To test the adjustment of the light system mercury drops were suspended between the plates of the condenser and photo-emissions were obtained when the instrument was set for λ 2536. As no emission could be obtained from the mercury for the adjustments of the spectroscope at wave-lengths shorter than λ 2536 due to their low intensity, the only conclusion that could be drawn from this work was that the long-wave limit for the sulphur was some wave-length shorter than λ 2536. It was next attempted to set some closer limits for the long-wave limit by using absorption screens.

After studying a number of filters Cobalt chloride in absolute methyl alcohol was found most suitable. An absorption cell with quartz windows about 8 mm. wide was used. Solutions of Cobalt chloride in this cell, varying in strength from 2 *N* to .01 *N* were studied photographically with a quartz spectrograph and a mercury vapor lamp. The greatest precautions to prevent stray light and fogging were taken and the exposure times were made as long as fogging permitted. These photographs showed the solutions to have a narrow absorption band covering the range λ 4600 to λ 5100 and complete absorption of radiations of wave-lengths shorter than λ 2650 for the 2 *N* solution, and of all radiations shorter than λ 2400 for the .01 *N* solution. The methyl alcohol cut off all radiations of wave-length shorter than λ 2350. With the quartz mercury lamp that was used, photographing directly and using every precaution lines were obtained down to λ 2150. These photographs were repeatedly taken and the sharpness and completeness of the absorption ranges well substantiated.

No emissions from the sulphur were obtained when the radiations passed through the cell filled with any of the cobalt chloride solutions or with the methyl alcohol only. Many drops were held under observation for more than an hour each with the methyl alcohol cell before the quartz window. After a drop was secured it was exposed to the direct radiations and several emissions observed then the cell would be inter-

posed and the emissions would immediately stop; at the end of 30 minutes the cell would be removed and emissions from the drop would begin immediately in every instance. This definitely placed the long-wave limit at some wave-length shorter than λ 2350.

An absorption screen was looked for that would transmit some frequencies to which the sulphur was photo-sensitive; some transparent commercial *fused* quartz was found which when interposed in the path of the radiation cut down the emissivity from 40 emissions to the hour to 5 an hour. These plates were about 5 mm. thick and only one was used. If a plate of crystal quartz of the same thickness was placed in the path of the radiation there was no reduction in the rate of emission. This was verified in a number of drops. On examining the transmission of this plate it was found to cut off (at least photographically) all wave-lengths shorter than λ 2250.

Some time after completing this work I noticed that Hughes in his book, Photo-Electricity, had remarked that a column of water 1 cm. long cut off all radiations shorter than λ 2200. This was confirmed photographically and as a check on the earlier work, the emission of sulphur through this filter was examined and its rate of emission was found to be decreased in about the same ratio as with the fused quartz plate filter.

The long-wave limit of photo-sensitiveness of sulphur can be placed with a great amount of certainty between λ 2400 and λ 2200 and it is probably within the narrower limits of λ 2350 and λ 2250.

V. SHELLAC.

Drops of shellac were obtained by atomizing a filtered solution of shellac flakes dissolved in ethyl alcohol. The alcohol evaporates rapidly and the density of the sphere reaches a constant value in a very few minutes, as is shown by the constancy of the gravity speeds. The drops showed perfect sphericity when examined under a microscope and showed none of the variations in speed on succeeding excursions under gravity as did sulphur.

All the sulphur was washed out of the large cylinder and removed from the condenser plates. The entire interior of the iron cylinder was coated with shellac and this allowed to stand several days after which the air was pumped out of the tank several times and the air that was admitted each time was filtered and dried. A thin coating of paraffine was used on the plates as with the sulphur. The shellac drops were much more satisfactory to work with as there was no evaporation, and little, if any, fatigue effect of photo-emission.

TABLE V.

Drop No. 49.

P.D. = 6,000 volts.
Time under obs. = 90 minutes.

t_1.	t_2.	$\left(1 \pm \frac{t_1}{t_2}\right)\frac{1}{V} \times 10^2$.	No Units on Drop.	No Electrons Emitted.	Time Interval Before Emission, Secs.
23.0	31.8*	.0465	1 Pos.		
	51.3*	.0920	2	1	20
	128.	.1367	3	1	90
	Inf.	.1666	4	1	10
	62.0	.2286	5	1	120
	35.6	.2745	6	1	40
	25.0	.3200	7	1	0
	18.2	.3775	8	1	80
	15.6	.4125	9	1	30
23.2	35.4	.2757	6 Neg.		
	61.0	.2296	5	1	15
	Inf.	.1666	4	1	40
	130.0*	.1370	3	1	65
	52.0*	.0926	2	1	5
	32.2*	.0466	1	1	10
	23.0*	.0000	0	1	12
	31.8*	.0448	1 Pos.	1	28
23.2	Inf.	.1666	4 Neg.		
	133.0*	.1375	3	1	68
	50.0*	.0895	2	1	70
	32.0*	.0462	1	1	10
	23.0*	.0000	0	1	25
23.4	32.2*	.0461	1 Pos.		
	52.0*	.0916	2	1	70
	134.0*	.1375	3	1	45
	Inf.	.1666	4	1	150
	62.5	.2290	5	1	10
	35.6	.2761	6	1	35
	25.4	.3200	7	1	40
23.4	32.5*	.0466	1 Pos.		
	52.4*	.0923	2	1	140
	133.0*	.1373	3	1	30
	Inf.	.1666	4	1	25
	62.0	.2295	5	1	100
	35.4	.2766	6	1	75
	25.2	.3211	7	1	5
	18.2	.3808	8	1	2

All t_2 marked with * the drop was going downward due to the force of gravity being greater than the oppositely directed force of the electric field. In these cases the minus sign is used in the formula $\left(1 \pm \frac{t_1}{t_2}\right)$.

With the complete radiation from the mercury vapor lamp the photo-current from the shellac drops was less than that obtained initially from the best sulphur drops. However there was much less variation in photo-emission from drop to drop than from the sulphur.

The time interval from the opening of the shutter until an emission had occurred was observed for about 200 emissions. This was taken when the excess positive or negative charge was never more than 5 units. The average time interval was 50 seconds, the maximum time was 4 minutes and the minimum was too small to be measured by a stop watch.

In the study of the valency of the emission data were taken on 40 drops. More than 800 emissions were observed and the number of electrons liberated at each emission determined. The same procedure was followed as in the case of the sulphur.

Table No. V. gives the data on a typical drop. The time interval given under " time of exposure " is the interval of time between the opening of the shutter and the occurence of the emission.

Table No. VI. contains a summary of data on 10 drops.

TABLE VI.

Drop. No.	Time Under Obs.	No. Emissions.	No. Singles.	No. Mult.
17	120	47	47	0
19	80	29	29	
20	100	28	28	
21	60	23	23	
27	120	44	44	
31	120	40	40	
37	60	19	19	
38	50	17	17	
40	70	26	26	
49	90	30	30	

As seen from the table every emission observed corresponded to the liberation of a single electron, this was also true of the observations on the remaining drops. The conclusion thus seems fully warranted that the process of photo-emission liberates one and only one electron from the molecule in sulphur and shellac. This is rather interesting especially in the case of shellac in which there are complex molecules made up of a large number of atoms.

As it is generally recognized that there is an absence of free electrons in non-conductors and insulators, the actual observation and identification of electrons emitted photo-electrically from them gives added weight to the accumulating evidence that photo-electrons come from the atom structure and are not free electrons.

The long-wave limit of the shellac was found to be quite definitely somewhat shorter than that of sulphur, although I believe not a great deal shorter. No emissions were obtained from a large number of drops when the radiations were passed through the methyl alcohol solution. A fused quartz plate was then interposed in the path of the beam and 10 drops were observed for intervals of time exceeding an hour and no emissions occurred, but immediately after the plate was removed emissions would begin quite normally. The filter of water one cm. in thickness was interposed in the path of the beam for each of the 10 drops and emission was entirely stopped, but immediately after removing the filter emission began in every case. The conclusion then seems fully warranted that the long-wave limit for shellac is shorter than λ 2200.

Allen in his book Photo-electricity, p. 83, in discussing W. Wilson's work says, "Although shellac itself is not photo-electrically active, it allows the photo-electric current to pass through it when a thin layer is laid upon a metal plate." As is evident from what has been given we found that shellac was photo-sensitive. The photo-sensitiveness of shellac in thin sheets was examined by covering the metal plates of the condenser with a thin coating. The photo-emission from the coated plates was stopped by the same filters that stopped the emission from the shellac spheres; although this was not found to be the case with the cleaned metal plates, which indicates that this emission is from the shellac and not from the plates.

VI. Paraffine and Oil.

Commercial paraffine was melted and a thin coating of it placed on the condenser plates in a portion of the work with shellac and sulphur because there was no emission from it when illuminated by the mercury vapor radiation direct.

The oil used throughout the work on the evaluation of "e" by Professor Millikan was examined for its photo-emission by suspending the drops in the path of the radiation and found not to be photo-sensitive to the radiations from the mercury light direct.

As only a small piece of fluorite just large enough for a window in the ebonite strip was available, a pair of zinc electrodes were placed just inside the inner cylinder and in the narrow space between the edge of the condenser plates and the wall, directly in front of the small fluorite window. By using a tuned circuit very much energy was put into the spark. When the spark was running it caused currents of air in the cylinder which made the drops drift badly.

Great precautions were taken in sealing the condenser so that its

interior would not be affected by outer currents. By doing this, drops of either oil or paraffine could be kept under observation for more than 30 minutes while the spark was running. The drops of paraffine were obtained by melting the paraffine in the atomizer used for the sulphur. The oil drops were obtained by atomizing in the usual method.

With the radiation from the zinc spark passing through a thin fluorite window and about 12 cm. of air photo-emissions were obtained from both the oil and the paraffine drops. Several emissions were obtained from each of 20 drops of both materials and no emissions were obtained from any drops of either material when radiations from the mercury lamp fell on the drops. The conclusion that both oil and paraffine are photo-sensitive and that their long-wave limit is shorter than λ 2150 (this is the shortest wave-length photographed through the mercury lamp used) seems fully warranted.

VII. Summary.

1. A method has been given for detecting very weak photo-emission currents from insulators. Photo-emission currents were observed from sulphur, shellac, oil and paraffine.

2. The long-wave limit for sulphur has been placed between λ 2400 and λ 2200.

3. The long-wave limit for shellac was found to be some wave-length shorter than λ 2200.

4. The long-wave limits for oil and paraffine were found to be below λ 2150.

5. The photo-emission from molecules of sulphur and shellac consist in the ejection of a single electron from the molecule at each emission.

This work was suggested by Professor Millikan and carried out under his direction. I desire to express my appreciation for his interest and helpful suggestions, also to Mr. V. H. Gottschalk who was associated with me in the preliminary work.

THE DETECTING EFFICIENCY OF THE ELECTRON TUBE AMPLIFIER.

BY E. O. HULBURT AND G. BREIT.

SYNOPSIS.

Definition of Detecting Efficiency.—The detecting efficiency of an amplifier is defined as $\lim_{A \pm 0} \frac{b_0}{A^2}$ where b_0 is the average change in the plate current of the last tube and A is the amplitude of the impressed grid voltage.

Relative Importance of Detecting Efficiency and Input Impedance.—A discussion of the necessity of taking into account both the detecting efficiency and the input impedance is given.

Measurements of Detecting Efficiency for a Transformer-Coupled Radio-Frequency Amplifier.—Measurements of the detecting efficiency of a transformer-coupled radio-frequency amplifier were made by means of a condenser potential divider and a sensitive quadrant electrometer.

Measurements of Amplification.—Measurements of the amplification due to each tube of the above amplifier were made. It was found that the *sound intensity* in the telephones was increased in the ratio of $9 \times 10^3 : 1$ owing to the use of the first two tubes.

1. INTRODUCTORY.

THE multi-stage electron tube amplifier such as is frequently used in radio practice and laboratory experiment is essentially an instrument for giving a relatively strong response, in the form of an electric current or a sound, to a relatively small alternating voltage impressed on its input terminals. It is of basic importance to be able to describe the behavior of an amplifier, or in other words, to be able to predict how a given amplifier will act under given conditions. The behavior of an amplifier depends upon the manner in which it reacts upon the external input circuit and upon the actions which take place inside itself. The general problem, then, of investigating the behavior of an amplifier requires the consideration of two interrelated problems, the first being the determination of the reaction between the amplifier and the external input circuit, the second being the determination of the action of the amplifier itself. The first we term the *problem of input impedance*, the second the *problem of detecting efficiency*.

To emphasize the importance of distinguishing between the two phases of the general problem let us consider a specific example. Suppose it were required to compare two different amplifiers. To do this it would seem sufficient to listen by means of them to a steady signal, connecting

each one in turn to the same circuit. Such a procedure, however, would not in general give a fair test. It may happen that the first of these amplifiers, say A, is equivalent to a capacity in series with a large resistance and gives a large change in the output plate current for a small input E.M.F.; while the second, say B, is equivalent to a capacity in series with a small resistance but gives a much smaller change than A in the output plate current for the same input voltage. If then the test were performed on a receiving circuit having a low resistance and capacity B would not influence the antenna current appreciably while A would decrease it on account of a large input resistance. The decrease might be so large that the rectification in the plate circuit might be less with A than with B. Thus the test would be in favor of B. If the same test were performed on a high resistance circuit the amplifiers being connected across a large capacity, then neither amplifier would affect the currents in the receiving circuit appreciably. Hence, since A gives a larger rectification for the same input voltage than B, the test will be in favor of A. It is thus seen that an intelligent choice between the two amplifiers cannot be made unless both phases of the problem have been solved.

2. THE INPUT IMPEDANCE PROBLEM.

Let us suppose that the grid and filament of the first tube of an amplifier are connected to a circuit in which high frequency current is flowing. This circuit is known as the input circuit. In general it is not legitimate to assume that the current in the input circuit is the same as it is if the amplifier were absent even though there may be no direct inductive coupling between the input circuit and the various internal circuits of the amplifier. The problem of unravelling the factors which enter into this effect is termed the input impedance problem.

This question has been discussed by H. W. Nichols[1] and by J. M. Miller.[2] They showed that the effect of a single tube, when the grid current is zero, is equivalent to that of an electric circuit having a definite resistance and reactance which was calculated. In a paper[3] by one of us the single electron tube has been dealt with for the cases of positive and negative grid voltage, and the exact formulas have been worked out.

Thus, the problem of the input impedance is considered to have received a complete theoretical solution for the case of a single tube. The more complicated case of the multistep amplifier has as yet not been treated theoretically, although it would appear that the problem offers no fundamental difficulty.

[1] PHYS. REV., 13, 404, 1919.

[2] Bureau of Standards Scientific Paper, No. 351.

[3] "The Calculation of Detecting and Amplifying Properties of an Electron Tube from its Static Characteristics," G. Breit. (As yet unpublished.)

3. THE DETECTING EFFICIENCY OF THE AMPLIFIER.

The main purpose of the present work was the consideration of the second phase of the general amplifier problem, namely, the detecting efficiency of the amplifier. The detecting efficiency of the amplifier means, in general words, the efficiency of the amplifier to make weak signals intelligible. A precise definition of detecting efficiency is given later on. From general considerations it can be seen that the detecting efficiency depends upon the relation between the input grid voltage change and the resulting change in the output plate current. Therefore this relation must be determined either by theory or experiment before much can be said about the detecting efficiency of the amplifier. To determine this relation from theoretical considerations is, however, a somewhat difficult matter. Let us point out some of the difficulties briefly. Since the relation between the input grid voltage change and the output plate current change depends upon the amplification and rectification taking place in each tube, it is necessary to have an accurate knowledge of the characteristics of the electron tubes. These characteristics must be known with a precision sufficient to enable one to compute the first and second derivatives of the plate current and grid current with respect to plate voltage and grid voltage, respectively. It is further necessary to know the resistance and reactance of all the electrical circuits connected to the tubes for all frequencies. If the tube and circuit constants have been ascertained, then the output plate current can be computed for a given input grid voltage. Since both the measurements of the constants and the computations in the case of the amplifier are somewhat involved it appeared better from an experimental standpoint to measure directly the output plate current as a function of input grid voltage.

4. GENERAL EXPERIMENTAL PROCEDURE.

The type of amplifier chosen for investigation was the three tube high

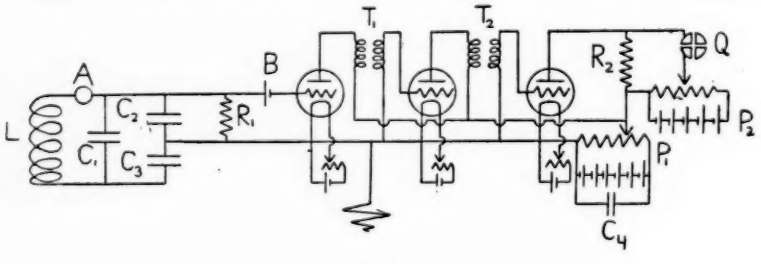

Fig. 1.

frequency transformer-coupled amplifier. This amplifier, shown schematically in Fig. 1, may be used to receive modulated radio frequency

signals. Apparatus was arranged to investigate the effect on the rectified component of the plate current of the last tube of impressing in some branch of the input grid circuit a high frequency E.M.F. of the form $A \cos \omega t$, where A is a constant. It is shown in the following paragraph that the results obtained from such a procedure are also true for the case where A is not a constant but varies in a certain manner.

Suppose that the frequency of modulation is so low that the reactances of the plate and grid circuits are negligible for that frequency. Let there be an external E.M.F. impressed in some branch of the circuit between F_1 and G_1 of the form

$$\varphi(\omega' t) \cdot \cos(\omega t - \epsilon),$$

where ω/π is the radio frequency and ω'/π is the frequency of modulation. φ is a periodic function of period 2π, which is supposed to be always positive for real values of the argument. If $\varphi(x)$ is zero for some real value of x the modulation will be said to be *complete*. If there is no real value of x which makes $\varphi(x)$ zero the modulation is said to be *incomplete*. Since the reactances of all the circuits are negligible at the frequency $\omega'/2\pi$, it is legitimate to assume that at any instant of time, t_0, the plate current of the last tube is the same as it would be if an E.M.F. $\varphi(\omega' t_0) \cos(\omega t - \epsilon)$ had been impressed on the amplifier for an infinite time. If such were the case, the plate current would in general be different for different values of $\varphi(\omega' t_0)$. Thus it is sufficient to investigate the effect on the plate circuit of the last tube of impressing in some branch of the input circuit an E.M.F. of the form $A \cos \omega t$ where A is a constant.

5. Experimental Details.

The apparatus consisted of a condenser potential divider, the amplifier, and a quadrant electrometer connected in the plate circuit of the last tube of the amplifier. These will be described in the order named. The arrangement is shown schematically in Fig. 1.

In order to impress on the grid of the amplifier a high frequency voltage of known amplitude of the same order of magnitude as that obtained in the reception of radio signals a condenser potential divider was devised. This consisted of three variable condensers of capacities C_1, C_2, and C_3 connected to a coil L in which high frequency current was induced by a suitable electron tube generating set. A thermo-galvanometer A measured the total current through the combination of condensers. The amplifier was connected across C_2, so that the voltage across C_2 was E_g, the effective grid voltage of the input tube of the amplifier. The high resistance leak R_1 (about 2 megohms) was con-

nected across C_2 to ensure a definite value of E_g during the experiment. The effect of R_1 upon the impedance of C_2 was negligible, because C_2 was large (about 0.05 μF) and the frequency used was of the order of 3×10^5. If the effective value of the current through A is I and the frequency of the current is $\omega/2\pi$, then it may be shown that

$$E_g = \frac{C_3 I}{\omega(C_1C_2 + C_1C_3 + C_2C_3)}.$$

By adjusting C_3 to a small value and C_2 to a large value the coefficient of I in the above equation may be made so small that a readable value of I is obtained when E_g is comparatively small. The absence of stray capacity effects was tested by using different condenser settings and different values of current. In order to eliminate direct action between the generating set and the amplifier, twisted leads about five meters in length were used between L and C_1, and between C_2 and the amplifier. This difficulty increases as the number of stages in the amplifier is increased, but by increasing the distance between the various parts of the apparatus it appears to be possible to reduce any direct action sufficiently for practical purposes.

The amplifier was a three-tube high-frequency transformer-coupled amplifier. The tubes were Western Electric Company tubes, Type VT1; they were used with the filament current always 1.10 amperes and the plate voltage always 22.0 volts. Separate storage cells supplied each filament; the plate voltage supply was common to all the tubes. By means of a standard cell B, Fig. 1, the input voltage E_g was kept always at a standard value. The plate battery was shunted by a $2\mu F$ condenser C_4. The transformers T_1 and T_2 were resonance transformers, both tuned to the same radio frequency. They were made of No. 36 silk-covered copper wire wound on paraffined wooden spools 3 cms. in diameter, 200 turns on the primary and 250 turns on the secondary winding.

The change in the value of the rectified component of the output plate current of the amplifier was measured by a Dolezalek quadrant electrometer Q, Fig. 1, connected across a high resistance R_2 placed in the plate circuit. The connections are shown in Fig. 1. The sensibility of the electrometer was 2500 mm. deflection per volt difference of potential between the quadrants. R_2 was 60,000 ohms. Therefore the electrometer deflection in millimeters could be reduced to the plate current change in amperes by dividing by 15×10^7. P_1 *and* P_2 are potential dividers, P_1 serving to keep the plate voltage at a standard value, and P_2 to compensate for the potential drop in the resistance R_2, so that the electrometer rested approximately at zero. When the input grid voltage

was changed, a deflection of the electrometer resulted which was proportional to the change in the rectified component of the output plate current.

It was important that the filament voltage of the last tube and the voltages of P_1 and P_2 be constant. Storage cells were found to be sufficiently steady for the purpose. When a slow drift of the electrometer occurred, the error was eliminated by averaging deflections.

6. Experimental Results.

The electrometer deflections, which were proportional to the change in the value of the rectified component of the output plate current, were recorded for a series of values of the amplitude of the radio frequency voltage impressed on the input grid for various frequencies. It was seen that the electrometer deflection was nearly proportional to the square of the amplitude of the grid voltage. This meant that the rectifying action of the last electron tube was represented approximately by the expansion of the plate-current grid-voltage relation, by Taylor's theorem, in which derivatives of higher order than the second were neglected. In Fig. 2 are shown the curves for the electrometer deflection plotted against the square of the grid voltage for the five different frequencies corresponding to the wave-lengths 800, 825, 850, 875, and 900 meters.

7. The Detecting Efficiency.

If A and b_0 denote the amplitude of the change in the input grid potential and in the rectified component of the output plate current, respectively, the detecting efficiency for a given frequency is defined conveniently by the relation

$$\text{detecting efficiency} = \lim_{A \doteq 0} \frac{b_0}{A^2}.$$

The detecting efficiency for a specified wave-length is obtained from the slope at the origin of the curve of Fig. 2 for that wave-length. The slopes at the origin have been computed for each curve of Fig. 2, and are shown in Fig. 3 plotted as ordinates against wave-lengths as abscissas. These slopes are reduced to the detecting efficiency by dividing by 15×10^7 which is the factor of proportionality between electrometer deflection in millimeters and the change in the rectified plate current in amperes. It is seen from Fig. 3 that the detecting efficiency of the amplifier is greatest at wave-length 850 meters. It is to be noted that the results shown in Fig. 3 give a complete solution of the question of the numerical value of the detecting efficiency of the amplifier. They do not, however, give any information as to the various factors upon which the detecting

efficiency depends. Nor do they yield information concerning the input impedance of the amplifier.

In order to check experimentally the assumptions of section 4 the following test was carried out. With a given current in A, Fig. 1, the

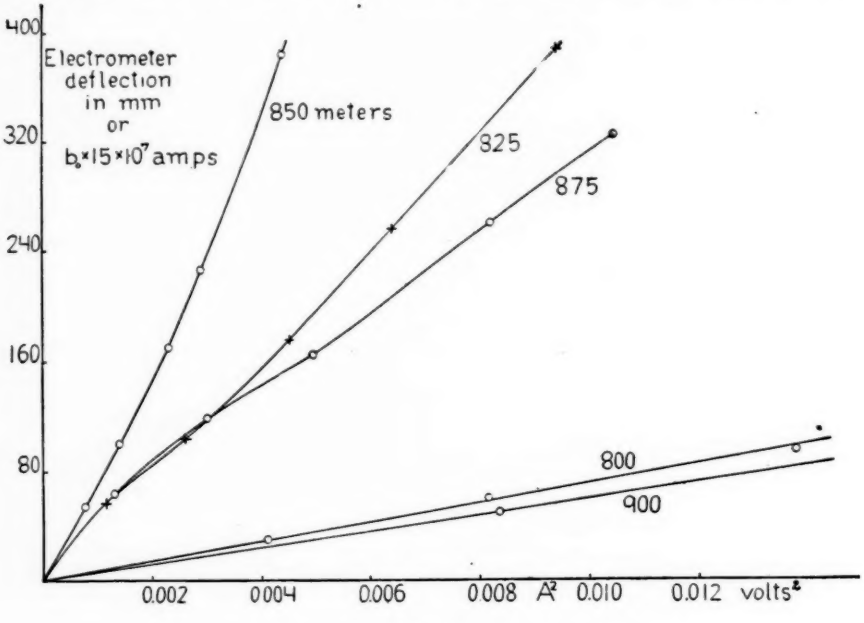

Fig. 2.

electrometer deflection was noted. A 500-cycle interrupter was introduced to chop the exciting current. When the coupling between the generating set and L was increased until the current had its previous

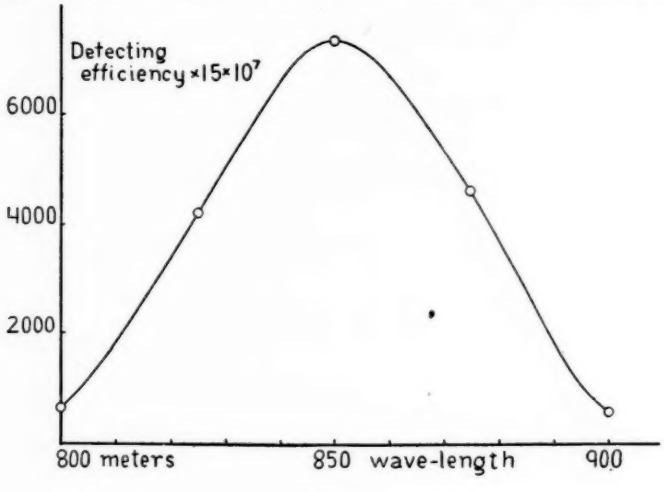

Fig. 3.

value, it was found that the electrometer deflection was the same as before. This test was repeated for various frequencies of the interrupter with the same result. This showed that modulating the exciting wave in the manner described above did not change the detecting efficiency of the amplifier.

8. Amplification of Each Stage.

The degree to which the detecting efficiency depended on each stage of the amplifier was determined by measuring the detecting efficiency of the amplifier for wave-length 850 meters with the first tube disconnected and then with the second tube also disconnected. It was found that the detecting efficiency of the amplifier with all three tubes was proportional to 9710, with two tubes 1020, and with one tube 104. The amplification due to the first tube was, therefore $\frac{9710}{1020} = 9.7$ and that due to the second tube was $\frac{1020}{104} = 9.8$.

This meant that the square of the radio frequency voltage from filament to grid of the second tube was 9.7 times as great as the square of the input grid voltage provided that the voltage was sufficiently small. Any rectifying action which occurred in the successive stages did not influence the result, because the slope of the curve was taken at the origin, and rectification is known to depend on terms in ΔE_g of higher order than the first. For amplitudes which can no longer be considered as infinitesimal the rectification may manifest itself by shifting the operating point on the characteristic of each tube and thus changing the amplification constant and internal resistance of the tube. This may be the explanation of the curious bends in the curves of Fig. 2.

9. Sound Intensity Amplification.

If a modulated radio frequency signal is received by means of the amplifier the kinetic energy of the vibrations of the telephone diaphragm, and also the energy of the sound waves produced thereby, is proportional to the square of the amplitude of the change in the rectified component of the plate current. The amplitude itself is, however, proportional for the same input radio frequency voltage to the square of the detecting efficiency as defined above. Consequently, for weak signals the sound intensity is proportional to the square of the detecting efficiency. In the case of the amplifier cited above the sound intensity was increased in the ratio of $9 \times 10^3 : 1$ owing to the use of the first two tubes.

Johns Hopkins University,
March, 1920.

THE MINIMUM ARCING VOLTAGE IN HELIUM.

BY K. T. COMPTON, E. G. LILLY, P. S. OLMSTEAD.

SYNOPSIS.

Helium Arc; Minimum Arcing Voltage.—The arc was stimulated by an intense thermionic current passed between electrodes in very pure helium. *20 volts* was found to be a well-defined minimum voltage at which the arc will strike or break, although it may be maintained on voltages as low as 8 volts after having struck.

Helium Spectrum; Voltage for Excitation of Lines and Bands.—Apparently the ordinary helium and parhelium lines and the bands are excited whenever the arc strikes. The line *4686* of the *enhanced system* was never observed below 55 volts, and was stronger above 80 volts. The lines of the *sharp subordinate series of pairs* are peculiar in that their intensity decreases, relatively to that of the rest of the spectrum, as the voltage is increased.

The results are in accord with *Bohr's theory of radiation and atomic structure.*

INTRODUCTION.

RECENT investigations of the production of radiation and ionization by electron impacts in helium[1] have shown that the minimum radiating potential of this gas is close to 20.2 volts and its minimum ionizing potential is 25.5 volts. One of the writers has shown[2] that these values apply to radiation and ionization set up by a single electron impact against a normal unexcited atom, whereas, if the electron current and gas density are relatively large, ionization may occur at any voltage above 20.2 volts. This ionization at abnormally low speed is presumably due to impacts against atoms which are in a relatively unstable condition due either to preceding impacts or to the absorption of radiation coming from neighboring atoms which have been struck. The latter of these causes is much more important than the former. Neither radiation nor ionization has ever been observed below 20 volts.

These considerations suggest that 20.2 volts should be the minimum voltage at which an arc can strike in helium and that the necessary conditions for obtaining an arc at this voltage are an intense bombarding electron current and relatively high gas pressure, so that the amount of ionization due to the comulative effect of impacts and radiation may be sufficient to cause an arc.

Rau[3] and Richardson and Bazzoni[4] have published results of experi-

[1] F. Horton and A. C. Davies, Roy. Soc. Proc. A., 95, p. 408, 1919; Franck and Knipping, Phys. Zeit., 20, p. 481, 1919.

[2] K. T. Compton, Phil. Mag. (in print)

[3] Sitz. Ber. d. Phys. Med. Ges. zu Würzburg, p. 20, 1914.

[4] Nature, 98, p. 5, 1916.

ments on arcing potentials and the arc spectrum in helium. The lowest arcing voltage found by Rau was 24.5 volts. This was in the presence of mercury vapor, whose ionization increased the current density and thus caused the helium arc to strike at a lower voltage than in pure helium. Apparently 29.5 volts is the lowest voltage at which Rau obtained the arc in pure helium. Richardson and Bazzoni, working with helium in the presence of mercury vapor, obtained the helium arc spectrum at 22.5 volts, the mercury arc spectrum appearing first at a slightly lower voltage.

The following experiments were made to determine the voltage at which the arc strikes under various conditions of gas pressure and electron current density in very pure helium. It was found that the arc could be made to strike at voltages as low as 20 volts, but never lower. Under favorable conditions, however, the arc could be maintained at much lower voltages, the lowest voltage observed being 8 volts, with a gas pressure of 5 mm. and a current of about an ampere through the gas. Observations of the spectrum were also made under various conditions.

Apparatus.

The arc was obtained between an incandescent tungsten wire cathode and a nickel disk anode, enclosed in a glass bulb. Wires of various diameters from 0.06 mm. to 0.25 mm. were used, and their lengths varied from 1 cm. to 2 cm. The distance between the electrodes varied between 1 cm. and 3 cm. in different bulbs. A large bulb of cocoanut charcoal was sealed directly to one end of the experimental tube, while to the other end was attached the glass tubing connection to the helium reservoir and pump. This connecting tube was bent into two U tubes near the bulb. The one nearest the bulb was a trap to prevent the entrance into the bulb of water or mercury vapors and the further one contained charcoal. The bulb and the charcoal tubes were baked out in an electric furnace maintained at a temperature between 300° and 350° C. for several days while the apparatus was evacuated by a diffusion pump. Liquid air was then applied to the trap and later to the two charcoal tubes before the helium gas was admitted. A mercury hand pump with a magnetically operated valve permitted the gas pressure to be adjusted to any value between 0 and 24 mm. A Hilger wave-length spectrometer was used for the examination of the spectrum. In every case observations were made at increasing filament temperatures until the wire burned out. The voltage across the arc and the current through it were regulated by series and parallel resistances connecting it to a 110 volt storage battery.

RESULTS.

From among the large number of sets of observations made only a few examples can be given in the accompanying figures. Possibly the results can be most concisely presented by discussing them under the four following cases, which are somewhat arbitrary and not always mutually exclusive.

Case I.—No arc. The current increases as ionization and radiation set in above 20 volts, but there is no discontinuous change or visible

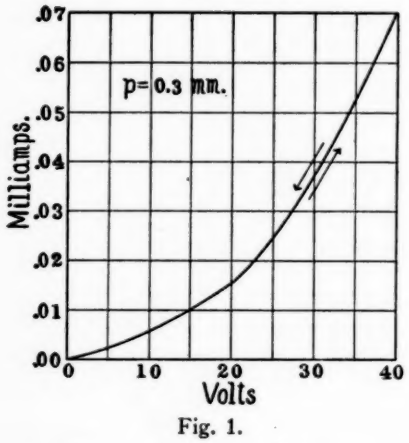

Fig. 1.

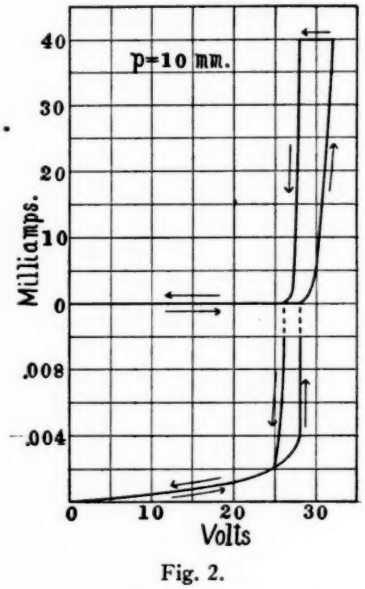

Fig. 2.

radiation. Variations of current with voltage are reversible. This case is observed at any gas pressure if the thermionic current is small and with any thermionic current if the gas pressure is small. Example: Fig. 1.

Case II.—The arc strikes at a voltage above about 29 volts and breaks at a voltage above 20 volts. There is no characteristic value for either the striking or breaking voltage, since they may be varied by varying the thermionic current or pressure. In general any change which increases one, also increases the other. The arc current always increases as the applied voltage is increased, but there is a region of irreversible changes between the striking and breaking voltages. Example: Fig. 2.

Case III.—The arc strikes at a voltage below 29 volts and breaks at 20 volts. Here it is found that, by increasing the thermionic current by raising the filament to a higher temperature, the striking voltage may be reduced as low as 20 volts, but the breaking voltage stays constant

at 20 volts. Between the striking and breaking voltages the current-voltage changes are irreversible. Example: Fig. 3.

Case IV.—The arc strikes at 20 volts and breaks at 20 volts, although it may be maintained at lower voltages if the current of the arc is increased by decreasing the series resistance. But no further increase in the thermionic current will cause the arc to strike or break at voltages below 20. This seems, therefore, to be a definite minimum value for the arcing voltage—and, therefore, for the excitation of the spectrum. If, after the arc has struck, the series resistance is decreased, the current increases and the potential drop across the arc decreases. In one case the arc was thus maintained on 8 volts and in another case on 10 volts. There seems to be no lower limit for the maintenance of the arc, once it has struck, for this seems to be limited only by the size of the currents which the electrodes will carry. In every such case, as the series resistance is again increased, the voltage again rises to 20, where the arc breaks. Example: Fig. 4.

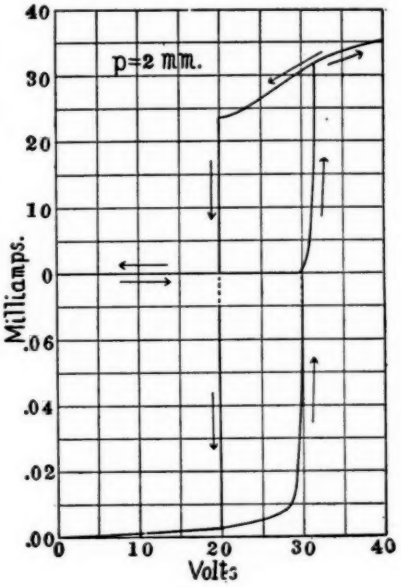

Fig. 3.

There are several interesting peculiarities of the arc which have been noticed during the experiments. Between Case III. and Case IV. there is an intermediate condition which may be considered as marking the division between them. Here the arc strikes at 20 volts, and remains at 20 volts however the current through the arc be increased by decreasing the series resistance. In this condition the resistance of the arc varies exactly inversely with the current through the arc. In one case the current between the electrodes was varied at will from 50 microamperes to 50,000 microamperes without producing any variation in the potential drop of 20 volts.

The current and voltage of the arc, with large currents, often exhibited discontinuous changes as the geometrical distribution of the arc in the tube suddenly changed. As far as we could tell, these shifts gave no indication of critical voltages in addition to that at 20 volts, since they seemed to be more or less accidental and occurred at different voltages at different pressures and with different positions and shapes of the electrodes.

It should be stated that the observed critical voltage which we have called 20 volts, in reality was sometimes as high as 20.5 volts and sometimes as low as 19 volts. These variations, however, are accounted for by the average initial velocities of emission of electrons from the filament at the temperatures used.

In securing an arc under the conditions of Case IV., the optimum gas pressure was between 4 mm. and 10 mm. Higher gas pressures would be advantageous, because of the greater probability of ionization by

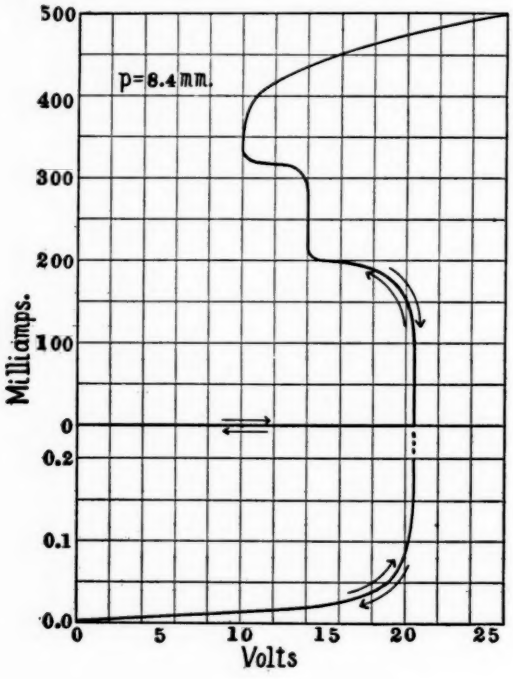

Fig. 4.

cumulative impacts, if it were not for the fact that, at such high pressures, the electrons make so many collisions in their zig-zag course that the aggregate amount of energy which they lose as a result of the momentum imparted to the atoms becomes considerable.[1] Thus, at the higher pressures, voltage greater than 20 volts must be applied in order that the electrons may acquire a velocity corresponding to 20 volts.

SPECTROSCOPIC OBSERVATIONS.

As far as we could tell, the entire helium spectrum, with the exception of the enhanced lines, appeared whenever the arc struck, and therefore

[1] Benade and Compton, PHYS. REV., 11, p. 184, 1918.

appeared at potential differences as low as 20 volts. This was apparently true of the band as well as of the line spectrum, although the relative faintness of the bands made it impossible to see them at voltages quite as low as 20. With the most intense arc, the lowest voltage at which the bands were observed was 26 volts, but the indications were that we missed them below this voltage because of lack of intensity rather than because they were not excited. The most prominent line of the enhanced spectrum is the first member 4686 of the series $4N[(1/3^2) - (1/m^2)]$. Rau was able to excite this line only with potential drops of 80 volts or more. At gas pressures of less than about 3 mm. and with thermionic currents not too intense, we have found that this line suddenly appears as the voltage is raised above 80 volts, but, with more intense currents and pressures between 3 mm. and 5 mm., the line may be seen at voltages as low as 55 volts. In these cases, however, the intensity of the line considerably increases if the voltage is raised above 80 volts. At still higher gas pressures, such as 10 mm., we were unable to excite this line at any voltage used.

We verified Rau's and Richardson and Bazzoni's observations that the line 4713, which is the second member of the sharp subordinate series of the system of pairs, decreased in intensity with increasing voltage, although the lines of the other series increased strongly. We observed this also in the case of the first member 7066 of the same series.

Although the band spectrum was excited at apparently the same voltage as the rest of the arc spectrum, it appeared that it was relatively most intense at high gas pressures, from 10 mm. up. At these pressures we observed the bands noted by Fowler[1] very easily, and, in addition, some fainter bands. One of these consists of two or more sections extending from about 5748 to 6044. Another seems to consist of six pairs, equally spaced, between 5719.5 and 5657. There is another set of fine lines beginning at about 4575 and extending to 4378, with evidence of the convergence of a series at the latter wave-length. We are somewhat hesitant about mentioning these, since our spectroscopic equipment was not sufficiently elaborate to permit of refined observations, and yet we are unable to attribute them to an impurity, since the gas seemed to be very pure. There was no trace of the spectra of mercury, other inert gases (with the possible exception of neon) or of hydrogen (except in the case of one bulb on which a quartz window was waxed and which could not be given adequate "baking out"). Several of the faint lines of these bands corresponded, within the limits of accuracy of our observations, with certain neon lines, but there was no trace of the three or four strongest neon lines in this region, or elsewhere in the spectrum, at pres-

[1] Roy. Soc. Proc. A, 91, p. 208, 1915.

sures at which the bands were prominent, and there was no uniformity between the neon spectrum in general and the observed spectrum.

DISCUSSION.

The results of this investigation confirm the view that ionization may occur at any voltage above the resonance potential as a result of the impact of an electron against an atom which is already partially ionized through the absorption of a quantum of resonance radiation from neighboring atoms. In other words, the presence of resonance radiation changes the condition of the atom to one of easier ionization. In the intense arcs in which the discharge was maintained far below 20 volts, it is evident that practically all the atoms were in an abnormal, easily ionizable, condition. We may picture the state of affairs as one in which an electron, returning toward its normal most stable position step by step, and radiating energy at each step, is again partially or entirely ejected towards less stable positions by absorption of radiant energy or by impact before it has an opportunity to return to the most stable position. Thus the bulb contains a large density of radiant energy of various spectral frequencies and helium atoms in all stages of ionization, or degrees of stability. The more intense the arc, the larger is the proportion of atoms whose electrons are in the outer orbits, or least stable configurations, so that the arc may be then operated on reduced voltage. The first step, however, essential to reaching this condition and the excitation of visible radiation, is the displacement of the electrons from their most stable to their next most stable configuration—a displacement requiring an amount of energy which is acquired by an electron in falling through 20 volts.

The investigations of low voltage arcs in mercury and other metallic vapors[1] are to be interpreted in a similar way. In every case the arc may be made to strike at a voltage equal to the resonance potential, and may be maintained at a lower voltage. Here, however, the voltages are low, so that the initial velocity distribution of the electrons from the filament introduces uncertainties of a relatively large magnitude as compared with those in the present case. By analogy with these metallic vapors, we should expect to find a very strong emission series in helium, with the first member (also an absorption line) at about 605 Å. and convergence at about 484 Å., corresponding to 20.2 volts and 25.5 volts.

The theories of atomic structure, which are rather uncertain with regard to the constitution and radiation of systems with more than one electron, have been quite successful in accounting for the spectra of

[1] McLennan, Phys. Soc. Lond. Proc., 31, p. 1, 1918; Hebb., PHYS. REV., 9, p. 372, 1917; 11, p. 170, 1918; 12, p. 482, 1918 and others.

atoms with a nucleus and a single electron. On Bohr's theory, the spectral series from helium atoms which have lost one electron should be given by the formulæ

$$4N\left(\frac{1}{n^2}-\frac{1}{m^2}\right),$$

whence the negative energies of the various configurations of an electron are given by $4N/n^2$, where n may have various integral values from 1 to ∞. Thus the energy required to change the electron from its most stable to its next most stable condition is $4N(1/1^2 - 1/2^2)$, or $3N$; while the energy required to completely remove it is $4N$. The minimum energy required to change the electron from the most stable condition to that condition in which it may emit the line 4686 is $4N(1/1^2 - 1/4^2)$, or $15N/4$. Thus the energy, in equivalent volts, required to remove the second electron from an atom already ionized, is 54.3 volts, while the energy required to cause such an atom to emit the 4686 line corresponds to 50.8 volts. Similarly, the minimum energy required to cause an atom, originally neutral and normal, to emit this radiation corresponds to $50.8 + 25.5 = 76.3$ volts; that required to cause an atom which has absorbed a quantum of the resonance radiation to emit this line corresponds to $76.3 - 20.2 = 56.1$ volts; that required to doubly ionize an atom at a single impact is $25.5 + 54.3 = 79.8$ volts if the atom is originally normal and is $79.8 - 20.2 = 59.6$ volts if the atom has absorbed resonance radiation.

There are thus various ways in which the 4686 radiation may be excited. Of these, the excitation by a single impact at 79.8 volts or more should predominate at low pressures and currents, while at higher pressures, with most of the atoms in the abnormal state, the excitation should be caused by 56.1 volts or possibly even by 50.8 volt impacts. These possibilities accord with our observations, in which the 4686 line was observed at voltages as low as 55 volts in a very intense arc, but with evidence of another method of excitation at 80 volts which was relatively more important at low pressures and arc intensities. The failure to observe the line at pressures above 10 mm. is evidently due to the very small chance of the electrons falling through a sufficiently large potential difference without losing their energy at an ionizing or radiating impact.

The fact that the lines of the so-called Parhelium series appear at the same voltages as those of the Helium series renders untenable Stark's conclusion that they are due to atoms which have lost more electrons than those atoms which give rise to the helium series lines.

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ELECTRIFICATION BY IMPACT.

BY HAROLD F. RICHARDS.

SYNOPSIS.

Electrification by Impact; Measurement of the Charge Produced by Collision between a Metal and a Dielectric.—After briefly discussing the unsuccessful attempts which have been made to formulate a satisfactory theory to explain electrification by friction, the author suggests that impact of dielectric upon metal, without sliding friction, may cause an electrical effect whose laws will shed light upon the frictional phenomenon. An apparatus is described for measuring the electric charge produced when a disc or sphere of dielectric material collides with a metal disc. The charges obtained in this manner ranged from 0.16 to 9.83 e.s.u., and produced potentials of 2.41 to 183.8 volts upon the metallic systems employed. These charges are of the same order of magnitude as those obtained by friction. The experiment was performed with *various metals and dielectrics*, and in every instance the metal received a positive charge. In no case was there any evidence of the erratic variation which others have found to be characteristic of electrification by friction. Curves are given which show the *variation of charge with velocity of impact* and with the *mass of the impinging system*. The charge produced by a single collision increases with each of these factors, but the velocity of impact was found to exert a greater influence than the mass of the moving body, in determining the amount of the charge. In certain cases velocities were attained at which the electrification due to a single impact reached a maximum value.

Relation of Charge to Capacity.—The quantity of charge produced by a given collision was shown to be independent of the capacity of the metallic system.

Effect of Repeated Impacts.—When many impacts were performed in rapid succession, the amount of charge increased to a maximum. This maximum was shown to be conditioned rather by the quantity of charge present upon the dielectric than by the potential of the metal anvil.

Discussion of Results.—The author concludes that there is no direct dependence of the electrical energy upon the mechanical energy lost in impact, and that electrification by impact is similar in nature to the contact effect between metals. The results are considered to support Helmholtz's theory regarding the nature of electrification by friction.

I. INTRODUCTION.

THE very meager quantitative results and theoretical study of electrification by friction have prevented the formulation of a satisfactory theory to explain the phenomena involved. Perhaps the most important work in this field is that of Owen,[1] who finds many indications that the effect is similar to the contact difference of potential between metals, thus agreeing with Helmholtz.[2] The latter had previously suggested that the function performed by the frictional work is

[1] Phil. Mag., XVII., p. 457 (1909).

[2] Wissenschaftliche Abhandlungen, Erster Band, p. 860.

merely to bring the molecules of the two substances into closer contact. Owen concludes that, with a sufficient amount of frictional work, the charge reaches a constant maximum, and that this maximum is attained with a smaller amount of work the greater the pressure between the rubbing surfaces. He notes further that less work is required to produce a given charge if contact is improved by a series of rubs. Jones,[1] however, definitely rejects these views, as is shown by the following quotation from his paper: "Frictional electricity appears, therefore, to be an effect of a different order from that of contact electricity, and it is worth while considering whether the facts cannot be accounted for on some other hypothesis." He assumes that the rate of production of charge is proportional to the rate at which work is performed, and explains the fact that the charge reaches a maximum value by assuming that back-leakage occurs in an amount proportional to the total charge present. It is difficult to see how this theory can account for the fact that one substance remains positive with respect to another, a phenomenon which is readily explained by the theory of Helmholtz. Thus, in spite of attempts to formulate a consistent theory, no great success has been attained, because the data of electrification by friction remain essentially erratic. This is abundantly shown by the experiments of Owen and Jones, and also by the later work of French[2] and McClelland and Power,[3] who find that not only the amount, but even the sign, of the charge varies with the conditions to which the rubbing surfaces are exposed before an experiment.

The present work was therefore undertaken to find what electrical effect would be produced by intimate contact, *without friction*, between metals and dielectrics. Both Owen and Jones have found that mere contact of such substances does not produce a detectable amount of electrification, but it is probable that the contact in their experiments was not such as would bring the surfaces sufficiently close together to furnish an adequate test. It was my belief that the more intimate contact produced by collision might cause an effect whose laws would shed light upon the nature of electrification by friction. It seems evident that the more definite nature of a single impact at a known velocity will give to the data of such an effect a greater consistency than that which characterizes the results of electrification by friction.

[1] Phil. Mag., XXIX., p. 272 (1915).
[2] PHYS. REV., IX., No. 2, p. 151 (1917).
[3] Roy. Irish Acad. Proc. 34, Sect. A, p. 40 (1918).

2. APPARATUS.

The apparatus used in the first experiments is indicated in Fig. 1. An ebonite disc E fits firmly into a brass socket attached to the lower end of a brass tube A sliding in a cylinder D. A is 2.5 cm. outside diameter, 17.5 cm. long, while D is 3.1 cm. inside diameter and 35 cm. long. The sliding tube is provided with studs to form a three-point contact at top and bottom, and these were carefully machined so as to permit free fall with minimum lateral freedom. Uniformity in bore of the stationary cylinder, the low-friction close-fitting contact of the bearing studs, and the relatively large ratio of the length of the falling

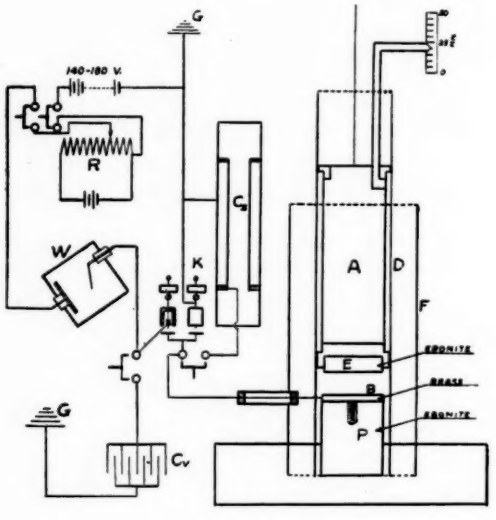

Fig. 1.

tube to its diameter, all combined to ensure uniform orientation of the ebonite surface at the moment of impact with a brass disc B. The latter was fastened securely to an ebonite pedestal P by means of a threaded projection of its own material. The pedestal was firmly held in a heavy iron base. All the horizontal surfaces were rendered parallel by careful turning in a lathe. An index rigidly attached to the plunging cylinder indicated the height of its fall. The air space between the cylinders provided windage, but windows were also constructed at the base of D to prevent compression of air. F is an earthed metal cylinder. The ebonite and brass discs were each 2.5 cm. in diameter, and of a thickness of 0.95 and 0.32 cm., respectively.

The charge produced upon the metal disc was measured by means of a Wilson tilted electroscope W, which was chosen on account of its low

capacity (2.3 cm.), and consequent high sensitivity to quantity of charge. Deflections were observed with a microscope containing a micrometer eyepiece. The plate of the Wilson electroscope was maintained at a constant potential of 140 to 180 volts by a battery of accumulators, and the sign of this potential was always kept the same as that of the charge to be measured, so that the instrument was used in its most stable condition. The scale of the microscope contained 40 divisions, and the position of the goldleaf could readily be estimated to 0.1 division. Calibrations made with multiples of Weston standard cells furnished curves showing the voltages corresponding to given deflections. Various sensitivities were used, ranging from 3 to 10 divisions per volt, and for these values the deflections were nearly, but not quite, proportional to the potential of the goldleaf. Accurate adjustment to the desired sensitivity was obtained by the use of a potentiometer arrangement, indicated at R. Under the conditions described, the Wilson electroscope furnished a most convenient and satisfactory means of measuring the charges produced by impact. The metal disc and measuring apparatus could be earthed or insulated either together or separately, by means of a double key K operated by a simple arrangement of two strings. Care to prevent leakage was taken and, in addition, all charged portions of the apparatus were housed in earthed metal boxes for electrostatic protection.

3. Measurement of Capacity.

The capacities used in these experiments were so small that they were measured by comparison with a specially-constructed condenser of concentric cylinders possessing a capacity of the same order. The dimensions of the inner tube were 2.70 cm. outside diameter and 43.15 cm. length; the outer cylinder was 4.45 cm. inside diameter and 52.50 cm. long. As the length compared to the space between the cylinders was large, the ordinary formula for capacity can be used. The calculated capacity proved to be 43.23 cm. (electrostatic units). All values of capacity used in the experiments were determined by comparison with this condenser. The capacity of the system was altered whenever necessary by inserting in the circuit an adjustable parallel-plate condenser, whose range of capacity was from 33.9 to 346.1 cm. The sliding plates of this condenser were earthed, and the plates receiving the charge were mounted rigidly on sulphur pedestals.

4. Experimental Results.

A. Variation of Charge with Velocity of Impact.

The surfaces of the brass and ebonite impact discs were made as flat and smooth as possible, and mounted so as to be parallel at the moment

of contact. The cylinder, carrying the ebonite disc, was loaded with lead shot to give a total weight to the falling system of 753.8 grams. The ebonite disc was raised to various heights by means of a string passing over a light pulley, and then released by a mechanical trip so as to fall freely upon the brass anvil. The index indicated that there was no rebound, and therefore only a single impact. The ebonite was then raised 20 cm. above the brass disc and the charge on the latter measured. Tests showed that the charged ebonite plunger was sufficiently distant to produce no appreciable inductive effect upon the measuring system. Between successive impacts the ebonite surface was discharged by means of radium. Velocities of impact were calculated from the observed distances of fall.

A single curve was obtained as follows: Three readings were taken at each selected velocity when the height of fall was successively increased, and then the run was repeated with decreasing velocities. The charge

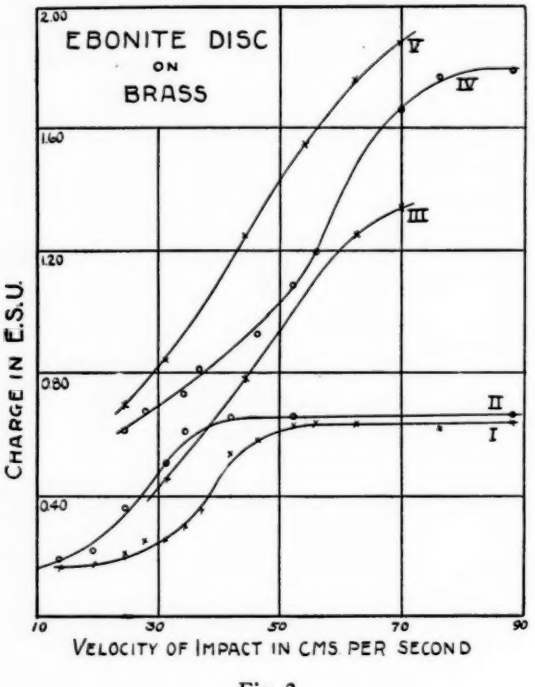

Fig. 2.

produced at a given velocity was then calculated from the six readings of the ascending and descending series, and this average was plotted. The charges obtained at a given velocity usually increased slightly after repeated impacts, probably due to further flattening of the surfaces, and

the effect of this small secular variation was eliminated by averaging together the ascending and descending runs. The usual variation from the mean in a series of observations at a single velocity was 3 per cent. No especial care was taken of the surfaces other than an occasional cleansing with alcohol. This treatment made only a small difference, usually increasing the charge by a small amount and rendering consecutive readings slightly more consistent. Upon one occasion cleaning the ebonite with a fine, flat file increased the charge from 0.54 to 0.99 e.s.u., but, in a velocity run immediately following, the charge reverted to its former value.

The curves exhibited in Fig. 2 show the charges obtained with a single impact at various velocities. These representative curves were selected from observations extending over a period of a month or more, during which many hundreds of collisions were produced between the same surfaces. It is seen at once that impact produces charges of the same order as those obtained by friction. These charges were of sufficient magnitude to raise the brass disc to fairly high potentials. Even the relatively low maximum charge shown in Curve II produced a potential of 9.75 volts upon the metallic system, whose capacity was 19.9 cm. (2.21×10^{-5} microfarads). The curves show a steady secular increase in the charge obtained at a given velocity, and also that, in the early work, when the surfaces were fresh, a maximum was attained at the velocities used, whereas no maximum was reached in the later experiments with the same surfaces. It may be supposed that this is due to increase in the contact-area, inasmuch as many collisions would render the ebonite surface more nearly plane.

It is worthy of note that the ebonite disc *always* produced a positive charge upon the brass anvil. There was no tendency towards the reversal of sign observed when charge is produced by friction. Even the slight residual charge obtained when the dielectric was lowered upon the metal with as nearly zero-velocity as possible was always positive, even when in amount it was not more than 0.02 e.s.u.

B. Relation of Charge to Capacity.

An exhaustive series of experiments was performed to find the relation between the capacity of the metallic system and the charge produced upon it by a given impact. The variable condenser was adjusted so that the charge due to a single collision gave a convenient deflection. Three readings were taken when the charge was formed at the capacity of the whole system, then three readings when the capacity of the measuring system was not added until after the impact had taken place. This

process was repeated a number of times in close succession, the charge being formed alternately upon the metal anvil and upon the whole system, but always measured at the same capacity. The velocity of impact remained constant throughout. Thus the only variable was the capacity at moment of production of charge. The mean charges found in four such experiments are given in Table I.

TABLE I.

Experiment No.	Velocity of Impact (cm./sec.).	Capacity at Impact (cm.).	Charge (e.s.u.).
1.....	67.1	{ 19.2 { 152.5	{ 1.42 { 1.43
2.....	62.6	{ 19.2 { 134.9	{ 1.98 { 1.97
3.....	70.0	{ 123.2 { 168.8	{ 2.22 { 2.23
4.....	62.6	{ ∞ { 168.8	{ 1.04 { 1.02

In order to employ infinite capacity the metal disc was earthed at the moment of impact, but insulated before the ebonite disc was raised. It had previously been found that the potential of the system was not altered until the dielectric was lifted.

These results show that the charge is independent of the capacity. On account of the important conclusions which can be drawn from this fact, the observations for one experiment are given (Table II.). Readings were taken alternately at the two capacities.

TABLE II.

Capacity at Impact.	Volts.						Mean Charge.
19.2.....	2.77	2.82	2.85	2.88	2.83	mean 2.83	1.42
152.5.....	2.78	2.84	2.85	2.91	2.87	mean 2.85	1.43

C. Influence of Mass of the Plunging System.

Fig. 3 shows velocity curves obtained in the same manner as previously described. The ordinates are the potentials produced by a single impact at the capacity of 133.3 cm. The run represented by curve *B* immediately followed *A*, the mass of the impinging system being decreased as indicated. Runs *C* and *E*, and *G* and *F*, were performed similarly. For the smaller mass, the charge is seen to be more nearly proportional to the velocity, and there is not the tendency to approach a maximum shown by the curves for greater mass. The curves show that a mean re-

duction in weight of 48 per cent. produced a mean decrease in charge of only 16 per cent., for constant velocity of impact.

D. Effect of Repeated Impacts.

Experiments were performed with repeated impacts in order to deter-

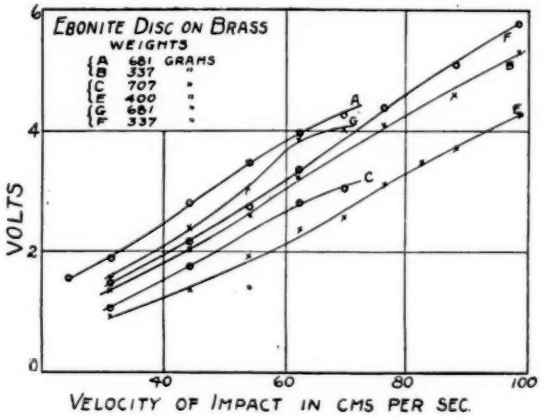

Fig. 3.

mine whether there is a maximum potential or a maximum charge obtainable by collision. Fig. 4, Curve I., shows the results when the

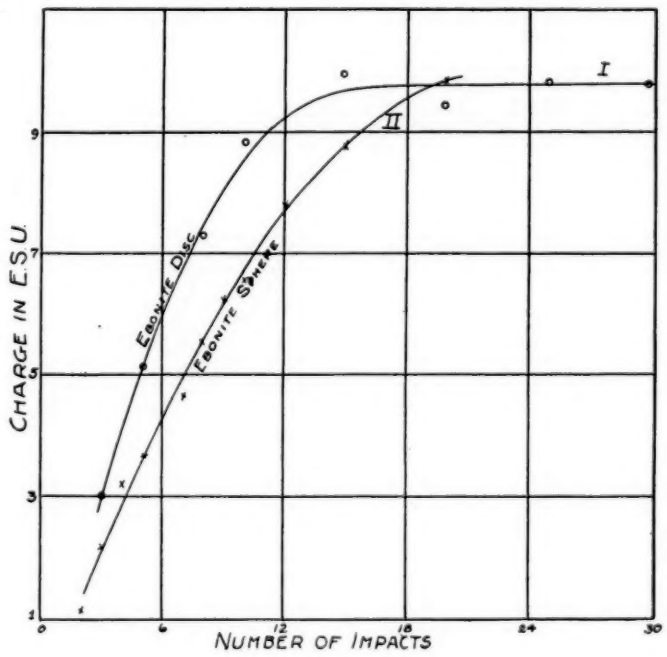

Fig. 4.

ebonite surface was not discharged between successive impacts. Fourteen collisions at a constant velocity of 62.6 cm./sec. could be produced in one minute. Insulation was excellent, so that leakage during the largest number of impacts was nearly negligible and could readily be allowed for by a small correction. The curve shows a maximum charge of 9.78 e.s.u., which produced a potential of 165.1 volts upon the metal disc. The charge was shared with a larger capacity for measurement. Then the ebonite disc, initially uncharged, was allowed to make five impacts upon the brass anvil, which was charged to various potentials by means of accumulators. The results are given in Table III.

TABLE III.

Velocity of Impact (cm./sec.).	Initial Potential of Brass Disc (Volts).	Charge Added by Five Impacts (e.s.u.).
62.6	280.2	3.31
62.6	350.2	3.42
62.6	386.0	3.09

There is seen to be no indication of a limiting potential, at the voltages used. It is thus evident that the limiting factor is not the potential of the metal anvil, but the amount of charge present upon the dielectric.

E. Effect of Static Pressure.

It has already been mentioned that a charge of approximately 0.02 e.s.u. was always produced when the ebonite disc was lowered upon the brass anvil with as nearly no velocity as possible. To determine whether greater pressure would increase this charge, weights ranging from 4.38 to 12.08 kilograms were placed upon the ebonite disc while it rested upon the brass. No increase in charge was observed.

5. THE HURLING EXPERIMENTS.

A. Apparatus.

In order to perform the experiments under widely different conditions, an apparatus was devised for projecting a sphere of dielectric upon an insulated metal disc. The construction is indicated in Fig. 5. A weight P falling upon the short end of a light, rigid oak lever L imparted a high velocity to a small brass tube H sliding in a heavy sleeve. The rise of the sliding tube was small, its motion being suddenly arrested by the abutments shown. The pole of the electromagnet M was provided with a collar A of non-magnetic material, in order to ensure uniform release of the projectile without lateral motion. The ball of dielectric rested in the smoothly beveled edge of the sliding tube. A calibration curve

was obtained showing the maximum free rise of the ball for different distances traversed by the falling weight. The action of this hurling device was constant to such a degree that the maximum variation from the mean in a series of flights was only 1.5 per cent. at the highest velocities used. The velocity of the ball at a given height h could readily be calculated fom the formula $v = \sqrt{2g(H - h)}$, where H is the maximum flight for the drop used. The ball could be projected vertically, so as to be pocketed in the hurling cup upon rebound, although in practice the ball was received upon heavy lead foil spread a little to one side of

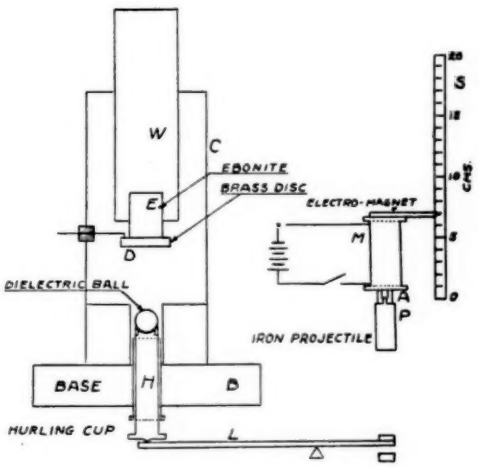

Fig. 5.

the sliding tube, and then rolled into the cup. The metal disc upon which the ball impinged was attached to an ebonite pedestal E inserted in a massive wood cylinder. C is a metal cylinder to afford electrostatic protection. The charge received by the disc was measured in the manner previously described. The capacity of the metal disc and its connecting wire was 16.0 cm.

B. Variation of Charge with Velocity.

Fig. 6 exhibits specimen curves showing the charge produced at various velocities by a single impact of ebonite upon brass. The dielectric sphere was 2.5 cm. in diameter and had a mass of 10.88 grams. The brass disc was 5.0 cm. in diameter by 0.32 cm. thick, and possessed a fairly high polish. The flight of the ball through the air before collision occurred was 11.6 cm. Discharge of the ball was effected before every impact by passing it rapidly through a flame, as it was found that the results were not affected by substitution of this agent for radium. Each

point on the curve is the mean of five or six observations at the velocity indicated. The maximum variation of a single reading from the mean was 8 per cent. As in the experiments with the plunging ebonite disc, the initial curves show that the charge reaches a maximum for certain velocities, whereas later work indicated that a much higher velocity would be required to produce a maximum. A possible explanation for this is that after a large number of impacts the ebonite surface was

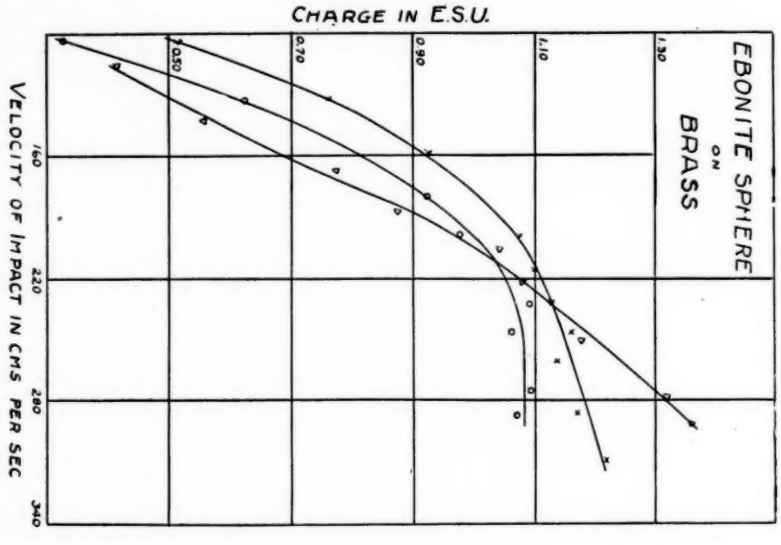

Fig. 6.

slightly bruised and flattened, although the brass showed no signs of wear. The surfaces of the dielectric and metal received no especial care other than an occasional cleansing with alcohol. No reversal of sign of the charge was ever observed; the charge on the brass was positive for all velocities and at every trial.

C. Repeated Impacts.

Fig. 4, Curve II., shows the charge produced by a given number of collisions at a velocity of 179.8 cm./sec. The ebonite ball was discharged between impacts. Seven collisions could be effected in one minute, and leakage during the time required for the largest number of impacts was nearly negligible, so that only a small correction was necessary. Twenty impacts produced a potential of 183.8 volts upon the brass disc. The curve shows that there is a tendency towards a maximum, although the dielectric was discharged between impacts. This result differs from the conclusion based upon the data given in Table III. for a similar experiment with the ebonite disc. The reasons for this will be discussed later.

D. Various Metals and Dielectrics.

Tests were made when a zinc disc was substituted for brass, and a glass ball for ebonite, the results of which are shown in Table IV. The charge is due to a single impact. The zinc disc was 5.0 cm. in diameter and 0.5 cm. thick. The glass sphere had a diameter of 1.9 cm. and a mass of 8.23 grams.

TABLE IV.

Materials.	Coefficient of Resiliency	Velocity of Impact (cm./sec.).	Charge (e.s.u.).
Ebonite on brass	0.74	292.0	1.37
Ebonite on zinc	0.70	166.8 to 252.8	0.87
Glass on brass	0.40	200.0 to 285.9	0.57
Glass on zinc	0.30	200.0 to 246.5	0.81

In every instance the metal received a positive charge. An ivory ball also produced a positive charge on brass. In all cases except that of ebonite on brass the metal was permanently indented by the impinging sphere, so that consistent velocity curves could not be obtained. For these cases the mean charge is given for a range of velocities at which the charge was approximately constant.

6. DISCUSSION OF RESULTS.

The results prove that intimate contact, without friction, between metals and dielectrics produces charges of the same order as those which were obtained by friction in the experiments previously mentioned. It is unlikely that any appreciable portion of the charge is due to friction. In both forms of the experiment, every precaution was taken to eliminate sliding contact of the surfaces. The lateral play of the ebonite disc was exceedingly small, and in the hurling experiments the sphere was projected vertically so as to avoid slipping of the surfaces. The hurling cup fitted the sphere evenly, so that the ball could hardly have possessed any considerable rotation at the instant of collision, and the time of contact was probably only a minute fraction of a second, since the speed of separation due to the large values of the velocity and resiliency was further augmented by gravity. In all published accounts of electrification by friction, repeated mention is made of the erratic character of the effect, which extends even to the sign of the charge. In the present experiments there was never the slightest tendency towards a change of sign, and the value of the charge remained remarkably constant during any one set of runs extending over several days, although it varied between fairly wide limits during the course of several months. In the

entire work there were not more than ten instances when the metal received a negative charge, and all of these occurred immediately after the specimen had been cleaned with alcohol, making it clear that, in these cases, the surfaces had not dried completely. The charge was always positive even when it was produced by a contact of almost no velocity. Any residual friction would in all probability have been extremely variable in amount, and this variation, superimposed upon the erratic effect of constant friction, would have seriously affected the consistency of results. Upon several occasions the ebonite disc was rotated by hand upon the brass anvil, which acquired a charge sometimes positive and sometimes negative. Sliding friction may therefore be completely dismissed in considering the results.

Apparently electrification by impact is a surface effect. Since the area of surfaces actually in contact increases with velocity of impact, greater charges can be expected for the higher velocities. This would explain the velocity curves obtained. The secular variation in the amount of charge may be accounted for by the fact that alteration in the shape of the surfaces occurs during lapse of time and repeated collisions.

It does not appear that there is any direct relation between the charge produced and the energy lost by collision. If Q and C represent, respectively, the charge and the capacity of the metallic system, and E the corresponding electrical energy, we may write $Q = \sqrt{2CE}$. If any constant fraction of the mechanical energy lost in impact at a given velocity were transformed into electrical energy, then E would be a constant for impact at that velocity, and the charge would vary directly as the square root of the capacity. Inasmuch as it has been shown that the charge is independent of the capacity, we may conclude that there is no direct dependence of the electrical energy upon the mechanical work performed in collision. As it seems probable that the frictional and impact effects are produced by the same mechanism, it is advisable to discuss the soundness of Jones's[1] conclusions. He states that the charge produced by a given amount of frictional work increases with the capacity of the metallic system. However, he does not mention the values of the capacities used, giving one dimension only for the cylinders employed to vary the capacity. This dimension he terms variously the length or the "thickness." Furthermore, his results are vitiated by a large secular variation, inasmuch as he found it necessary to perform complete runs at each of the different capacities in succession, whereas in the present experiments many readings could be taken alternately at the various capacities in a short time, so that not only would the

[1] Loc. cit.

secular change be small, but it could be eliminated by taking an average value. In this connection it may be noted that the recent work of McClelland and Power[1] contradicts Jones's conclusions that the maximum charge is independent of the temperature and velocity of the rubbing surfaces.

Further evidence in support of the view that there is no direct relation between the mechanical energy and the charge produced is furnished by the curves in Fig. 3. If, for a given velocity of impact, there were a direct proportionality between the electrical and mechanical energies, then in the expression given above, $Q = \sqrt{2CE}$, E would vary directly as the mass of the plunging system. Thus a reduction of 28 per cent. in the charge might be expected when the mass is reduced by 48 per cent. The actual decrease in the charge, however, was only 16 per cent. Additional evidence is afforded by the wide variation in mechanical energy lost per unit charge. For the ebonite ball, using the mean charge (Fig. 6) produced at a velocity of 295 cm./sec., the coefficient of resiliency being 0.74, we find 1.76×10^5 ergs lost per e.s.u. of charge. A similar calculation for the ebonite disc (Fig. 2) gives 1.50×10^6 ergs per e.s.u. Owen[2] finds that for ebonite rubbed with copper the same ratio has values ranging between 4.3×10^6 and 186.0×10^6.

So far as conservation of energy is concerned, the total electrical energy acquired is compensated by the mechanical work done in separating the charged surfaces. This is apparent at once from the consideration that at the moment the charge is produced the layers of positive and negative electricity almost coincide and therefore the potential of the system is not appreciably raised. No deflection of the goldleaf occurs until the surfaces are separated. However, the failure of the charge to increase when additional pressure is added, unaccompanied by a sudden impact, indicates that a certain amount of work is required to effect the passage of electrons from the metal to the dielectric, although it is possible that static pressures much greater than those used in the present experiments would increase the electrification. In this connection it should be recalled that, in the photoelectric effect, work is done by the electrons in emerging from the surface film. But in the present case so small a portion of the energy lost in impact is utilized in this manner that it cannot be stated how much work is necessary to produce the charge. Probably in the case of two optically flat surfaces the velocity of impact necessary to produce a given charge would be much smaller than in the present experiments.

[1] Loc. cit.

[2] Loc. cit.

The results for repeated impacts with the ebonite disc (Fig. 4) show a limiting charge of 9.78 e.s.u. Some view such as that of Helmholtz, who regarded the frictional effect as similar to the contact difference of potential between metals, must be assumed to account for the direction of the effect. From this point of view a very high maximum potential of the metal might be expected, on account of the vastly greater concentration of free electrons in metals than in dielectrics. This is borne out by the experiments in which an ebonite disc impinged upon metal surfaces at various potentials (Table III.), the charge produced showing no tendency towards a maximum. We may conclude, therefore, that the maximum is conditioned rather by the amount of charge present upon the dielectric than by the potential of the metal anvil.

In the experiment of repeated impacts with the ebonite ball, the latter was discharged between collisions. The curve (Fig. 4) shows that in this case there seems to be a tendency for the positive charge on the metal to leak back to the dielectric. This is at variance with the results mentioned in the preceding paragraph. This tendency towards back-leakage, however, might be greater for the sphere than for the disc, on account of the greater intimacy which higher velocities and curvature of surface would cause. The dissipation of energy per unit area is probably greater in the case of the sphere than of the disc, and the heat developed might also facilitate back-leakage.

It is a pleasure to acknowledge here many valuable suggestions received from Professor L. T. More and Professor R. C. Gowdy during the course of the experiments.

UNIVERSITY OF CINCINNATI,
February 11, 1920.

A NEW DESIGN OF PRECISION X-RAY SPECTROMETER.

By C. D. Cooksey.

Synopsis.

Principle of the Spectrometer.—The spectrometer is especially designed to make use of the "*Method of Displacement*," already published, for the determination of X-ray wave-lengths. This method is briefly described.

Description of the Spectrometer.—A detailed description of the spectrometer with a plan and side elevation are given.

Adjustments.—The processes involved in the adjustment of the spectrometer for precision measurements on X-ray wave-lengths are explained in detail.

Verification of the Fundamental Assumption on which the "Method of Displacement" is Based.—The method requires that a portion of the length of a beam of X-rays, after reflection, shall be of constant width. The possibility of this has been previously demonstrated on theoretical grounds. An exploration of the reflected beam for the α_1 line of the K series of silver has been made and the results are found to agree with theory.

Principle.

A METHOD has been devised for measuring wave-lengths in the X-ray region of the spectrum, by Professor H. S. Uhler,[1] which promises to eliminate many of the errors involved in earlier methods. The method requires the use of two slits of equal width between the source of rays and the crystal and a means of displacing the photographic plate in a straight line parallel to the line of collimation of the spectrometer, the plate undergoing no rotation during the displacement. The glancing angle of reflection at the crystal is then determined solely by the distance apart of the spectral lines, taken on both sides of the direct image of the slits in the two positions of the plate, and the distance through which the plate has been displaced. These distances can both be measured on the same comparator, thus making the determination of glancing angles depend solely on two linear measurements in the same units.

This method of displacement for determining glancing angles was given a preliminary trial by Uhler and myself[2] in our work on the K series of gallium. The spectrometer we then used had not been constructed originally for this method, but the results obtained with the remodelled apparatus encouraged us to believe that with a properly designed instrument the method would be capable of great precision.

[1] Uhler, Phys. Rev., N.S., Vol. XI., No. 1, p. 1, 1918.

[2] Uhler and Cooksey, Phys. Rev., N.S., Vol. X., No. 6, p. 645, 1917.

DESCRIPTION.

The spectrometer is shown in plan and elevation in Figs. 1 and 2. The main part of the spectrometer consists of a heavy cast iron base A, in the form of a rectangle, about 78 cm. long by 29 cm. wide. At the center of one end of this base is a vertical bracket carrying a vertical slit, S_1, with adjustable gold jaws 2.4 mm. thick. The base A is provided on one side with a V rail and on the other with a plane surface running parallel to the long axis of the rectangle. Another rectangular base, L, having a foot with a V groove at each end of one side, and a foot with a plane bottom at the middle of the other side, can slide on the rails of base A toward or away from S_1. A Société Genevoise circular dividing engine, without its knife carriage, is bolted on the upper surface of L in such a manner that the axis of rotation of the revolving table is parallel to the long axis of S_1. The normal from the center of S_1 to the axis of rotation forms the collimating line of the spectrometer. A second slit, S_2, similar to S_1, is fastened to the base of the dividing engine

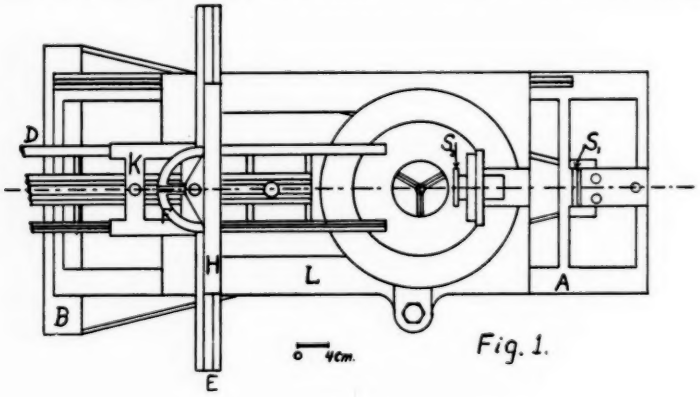

Fig. 1.

between its axis of rotation and S_1, and at the same height as S_1. S_2 is normally at a distance of 4.5 cm. from the axis of rotation, but this distance can be altered by a few centimeters. S_2 is provided with a slow motion adjustment by which it can be moved horizontally at right angles to the line of collimation. Another adjustment permits its rotation about this line or a parallel line as axis.

The face-plate of the dividing engine has fastened at its center a brass plate with three V grooves 120° apart, radiating from the axis of rotation. Two stands, not shown in the figures, are provided with three legs each to fit into these grooves. One of these stands has fastened to it the crystal holder. This holder has two horizontal linear adjustments, one parallel to, and the other at right angles to, the crystal face. A third

adjustment permits the crystal to be rotated about a horizontal axis parallel to its face. The other stand supports a slit, S_3, used only in adjusting the spectrometer. This slit is of fixed width of about 0.02 mm., and the jaws of the slit, when in place, do not come above the collimating line. Thus this slit does not limit those rays coming through S_1 and S_2 which are directed above the collimating line when they reach the third slit. S_3 is provided with adjustments by which the center of the upper end of the opening between the jaws can be made to remain accurately in the axis of rotation while the divided circle is turned through 360°. This adjustment was made under a high-power microscope, provided with cross hairs, and the fact that it is possible to so adjust the slit that no motion of the point referred to can be detected when the divided circle is rotated shows that the axle and bearings of the dividing engine are very true.

On the dividing engine base, diametrically opposite S_2 is mounted a pair of rails, D, about 42 cm. long and 10 cm. between centers. The casting carrying these rails is pivoted on the engine base so that the rails can be made parallel to the line of collimation. One of these rails is

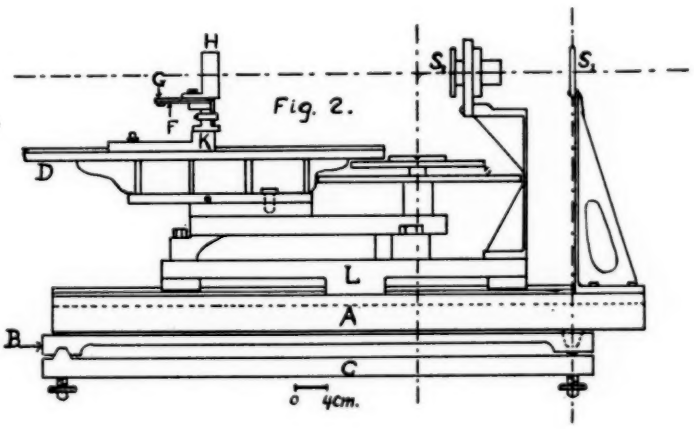

Fig. 2.

V-shaped in section, and the other square, with its upper surface plane. They were very carefully machined and ground smooth.

A carriage, K, fits on these rails and is furnished with a clamping screw to lock it tight to them at any point of their length. The extreme distance of this carriage from the axis of rotation of the spectrometer can be limited by an adjustable stop clamped to one of the rails, or by a rod of known length placed between the stop and the end of the carriage. It is this feature which permits the use of the "Method of Displacement"

already referred to. The end of the carriage nearer S_2 has fastened to it a horizontal aluminium rail, E, 42 cm. long, with its long axis at right angles to the direction of motion of the carriage. The transverse section of this rail is an inverted T. A horizontal brass semicircle, F, with a flange projecting vertically downward can be clamped on the T rail at any point. A flat brass plate, G, rests on top of the semicircle and is pivoted at its center. It can be rotated on the semicircle and clamped at any desired angle, F and G together forming a sort of protractor.

The plateholder, H, is a cast aluminium rectangular box, with plane surfaces inside, against which the plate can be pressed flat by means of strong springs on a wooden cover at the back. To allow for variations in the thickness of plates, the cover is lined with thick felt on the inside. A rectangular opening about 24 cm. long by 1.5 cm. high is cut in the front of the box and covered with black paper. The paper can easily be removed to permit a view through the plateholder or the measurement of the distance from the plate to the axis of rotation. The plateholder accommodates plates 25.4 cm. by 3 cm., the latter figure being determined by the dimensions of the device for holding the plates on the comparator used for measuring them. The central portion of the bottom of the casting extends behind the plane of the back of the rectangle in the form of a tongue, the bottom of which was machined flat. This tongue is provided with two dowel pins and a screw by which the plateholder can be screwed tight to the azimuth plate, G.

ADJUSTMENTS.

The object of this method of mounting the plateholder is to be able to photograph several spectral lines at a long distance from the direct beam without having to use excessively long plates and to be able to adjust the plane of the plate so that it may be either at right angles to the collimating line, or to the beam reflected from the crystal. A very convenient use of the aluminium T rail is in taking what I call "Range Plates." In using two very narrow slits it is necessary to know very closely what setting of the crystal gives the best reflection for a particular wave-length. To ascertain this a series of exposures of equal length are taken on the same plate, the plateholder being displaced a small distance along the T rail between each exposure, and the crystal setting altered a small amount. A comparison of these exposures shows, not only the best setting of the crystal, but, if enough are taken, the effective range of reflection of the crystal for the particular width of slits chosen.

The object of mounting the plateholder, crystal, and S_2 on a separate base from that on which S_1 is mounted is to be able to conveniently

vary the distance between S_1 and the axis of rotation of the spectrometer, which at the same time would, of course, vary the distance between S_1 and S_2. The distance of S_1 from the axis can be varied from about 11 cm. to about 35 cm.

The base A of the spectrometer is pivoted on a triangular cast iron base, B, in such a manner that A can be given a small rotation about the long axis of S_1 as axis. The base B rests, in turn, on a similar base, C, provided with ways on which B can slide a short distance at right angles to the collimating line. The base C is supported on leveling screws. Finally the table of the spectrometer can be rotated back and forth through a small angle by means of a spring motor connected to the tangent screw of the dividing engine through a connecting rod and crank.

The object of this rather elaborate mounting is to facilitate the lining up of the spectrometer with the focal spot on the anti-cathode of the X-ray tube. In working with soft X-rays it is necessary to have a window in the tube which will not absorb too much of the energy. Thin aluminium is very good for this purpose, but, when very thin, the window must be quite small, thus giving an emergent pencil of rays of small angular width. Experience has taught me that it is quite difficult, without special means, to make the collimating axis of the spectrometer coincide with the axis of this pencil. This adjustment is simple with the mounting described above. The instrument is first brought roughly into line with the aid of a fluorescent screen. A plate is then placed immediately behind S_1 and two exposures taken on it; a long exposure with the slit narrow and a short one with the slit wide open or removed entirely. This gives the projected images of the slit and the aluminium window superposed on each other. The center of the image of the window is then marked with a fine scratch, and the plate replaced in the position in which it was when the exposures were taken. By focusing the cross hairs of a microscope on the center of the image of the window, and moving the spectrometer across base C till the image of the slit comes under the cross hairs, S_1 is brought into the axis of the pencil of X-rays. In order to bring the collimating line of the spectrometer in coincidence with the axis of the pencil, S_3 is placed in position on the spectrometer table, and a plate placed immediately behind it. An exposure is then taken with S_1 narrow, and, if the lineup is badly out, another with S_1 wide open, and a screen placed directly above S_3. This gives two images, one above the other; the center of the upper one gives the position of the axis of the pencil, while the center of the lower one gives the position of the axis of rotation of the spectrometer with respect to the former. The plate is replaced on the spectrometer, and a microscope

focused on the center of the upper image. The spectrometer is then rotated on base B, about S_1 as an axis, till the image of S_3 comes under the cross hairs. The axis of the pencil of rays coming through the window on the tube will then pass through S_1 and the axis of rotation of the spectrometer. This being done, a similar method of superposed exposures will show the position of S_2 with respect to the collimating line, and any error can be corrected by means of the transverse adjustment referred to above. By means of a pair of parallels temporarily supported on the divided circle and extending between S_1 and S_2, the plateholder can be placed first in front of one slit, and then in front of the other without any rotation about the normal to the plate. Images of both slits can be taken close together on the same plate and any lack of parallelism between the slits corrected by means of the rotational adjustment on S_2 referred to above.

The rails D are made parallel to the collimating line by making S_1 and S_2 very narrow, and taking one exposure on the upper half of the plate when it is directly over the bearing on which the rails are pivoted, and a second on the lower half of the plate when it is at the extreme end of the rails. The rails can then be rotated so that their extreme end moves through the distance between these images. The straightness of the rails was tested by covering the plate with a screen having a narrow horizontal slit, and taking exposures with S_1 and S_2 narrow and the plate at different positions on the rails, the horizontal slit being moved vertically by its own width between each exposure. At the time of writing, the rails have been found to be very true, though there seems to be a very slight transverse shift of the plate when moved between certain positions. This may be due to dirt, but if it persists in future tests it can easily be measured and allowed for in determining wave-lengths by the "Method of Displacement."

The azimuth angle between the normal to the plane of the plate and the collimating line is completely determined by the difference in distance between the same spectral line photographed on each side of the central beam in two different positions of the plate on the rails D, and the distance between these positions. This angle can be made zero by rotating the plate G on the circle F or can be applied as a correction in the calculation of wave-lengths.

If the plane of the crystal is not exactly parallel to the axis of rotation of the spectrometer, the lines taken on either side of the central beam will be inclined to each other by four times the angle between the plane of the crystal and the axis. If two plates are taken at the same distance from the axis of rotation, each with the same line on both sides of the

central beam, and are then placed with their gelatin sides together, and the top of one at the bottom of the other, in such a manner that the corresponding lines on one side of the center are superposed, then the corresponding lines on the other side will be inclined to each other by eight times the angle of tilt of the crystal. This makes a very delicate method of detecting any error in the adjustment of the crystal face.

VERIFICATION OF THE THEORY.

It has been shown theoretically by Uhler[1] that a certain portion of a beam of monochromatic X-rays passing through two slits of equal width and reflected from a crystal is of a rectangular cross-section in a plane normal to the mutually parallel long axes of the slits, the length of the rectangular portion being equal to the distance between the slits, and the width being equal to the width of the slits. I have explored this rectangular portion of the beam in the case of the α_1 line of the K series of silver. The distances from the axis of rotation to S_1 and S_2 were 20 cm. and 4.5 cm. respectively, thus making the rectangular portion of the reflected beam 15.5 cm. long. Exposures were taken on the same plate at both ends of the rectangle and at points one quarter, one half, and three quarters of its length. This was done with both slits about 0.02 mm. wide and again with them 0.1 mm. wide. During the exposures the crystal was rotated back and forth through an angle of 3′ about the position which gave the best reflection.

The results of these photographs are in good agreement with Uhler's deduction that the portion of the reflected beam between the foci of the slits is rectangular in cross-section and of the same width as the slits. The exposures with the wider slits were purposely made short that they might bring out any lack of uniformity in the distribution of energy across the width of the beam at the different parts of its length. The results showed that the energy was fairly uniformly distributed and that, if the lines were thoroughly exposed, there should be no danger of an asymmetric blackening with respect to their axes.

Though this spectrometer has not yet been used for the direct determination of wave-lengths, every indication points to its giving a high degree of precision in such work. It should be pointed out here that there is nothing about the construction of the instrument to prevent its being used in the ordinary way for determining glancing angles by measuring the distance of the plate from the axis of rotation. Indeed, a determination of the glancing angle by both methods might give a measure of the depth of the mean reflecting plane of the crystal.

[1] *Loc. cit.*

In conclusion I wish to express my thanks to Professor Uhler for his many helpful suggestions and advice in connection with the design and adjusting of this instrument and especially for the plateholder and its adjustable mounting which were designed by him.

SLOANE PHYSICAL LABORATORY,
YALE UNIVERSITY,
March 15, 1920.

THE PERFORMANCE OF CONICAL HORNS.

By G. W. Stewart.

Synopsis.

The experiments were performed to secure quantitative results with a conical horn used as a receiver. The approximate theory of A. G. Webster, when extended, and the experimental data herein presented lead to the following conclusions concerning such a horn acting as a receiver:

Intensity at Vertex as a Function of Frequency.—The experimental variations of intensity at the vertex plotted with changing frequency, intensity of the arriving waves constant, demonstrate that:

(*a*) The frequencies producing resonance are very closely integral multiples of the fundamental frequency.

(*b*) The resonance peaks broaden and the minima between the peaks increase with increasing frequency.

(*c*) Therefore the conical horn used as a receiver gives an amplification for any frequency nearly the same or greater than the fundamental.

(*d*) The greater the frequency, the less the differences between maxima and minima, and thus a horn long compared to the wave-length of the least frequency involved, gives an amplification without highly marked resonance characteristics.

(*e*) The function of frequency obtained experimentally varies with the horn angle chosen, but retains the same general character.

Optimum Horn Angle for Fundamental Resonance.—There is a horn angle for which the amplification will be a maximum, assuming fundamental resonance. The optimum angle is greater the higher the frequency, 256 and 512 d.v. being used.

Optimum Horn Angle; Overtones.—Using 512 d.v. as the fundamental and first and second overtone, similarly 256 d.v. as the fundamental and first overtone, the optimum horn angle decreases with the order of the harmonic, being greatest for the fundamental.

End Correction.—The end correction of the conical horn is approximately 0.7 of the radius.

Conical Attachments.—Cylindrical and conical attachments to a conical horn introduce their own resonance characteristics, which may be estimated as to frequency by treating the end attached as a free open end but applying a positive correction to the length if the angle of the attached tube is less, and a negative correction if greater.

Limitations of Conclusions.—The conclusions apply strictly to the limits of frequency used in the experiment. Nevertheless they make more clear the nature of the action for all frequencies.

THE published theory[1] of the action of conical horns and pipes is unsatisfactory in at least two respects, namely, there is a masking of certain physical aspects of the action through the assumption of spherical waves, and there exists too great an inaccuracy in these com-

[1] See Rayleigh's Theory of Sound, Vol. II., Ch. XIV., and Barton, Phil. Mag., Series 5, Vol. 15, 1908, pp. 69–81, and Webster, Proc. Nat'l. Acad. of Sciences, Vol. 5, p. 275, 1919.

puted quantitative relations which should enable one to antitipate the amplification produced by a conical horn when used as a receiver. The present contribution furnishes certain quantitative information concerning the action of conical horns as receivers, under conditions as specified, and also gives a brief explanatory discussion expressed in terms of the physical action involved.

I. Apparatus and Methods.

Sources of Sound.—The sources of sound were two electrically driven tuning forks mounted upon wooden box resonators having openings of 10 x 5 cm. for the 256 fork and 7.3 x 3.7 cm. for the 512 fork.

General Method of Measurement.—A Rayleigh disc, connected by rubber tubing to the conical horn investigated, was used to obtain the relative intensities of the sound in the conical horn. The internal diameter of the rubber tubing was .45 cm. and the length about 150 cm. Readings were taken by the means of a telescope and scale. The deflections of the disc were proportional to the intensities in the disc and hence to the intensities at the vertices of the horns. A disc was constructed for each frequency, the upper one shown in Fig. 1 being for the 512 fork.

Location.—The source of sound, horn and disc were mounted on the edge of the roof of the physical laboratory 21 meters high, where the

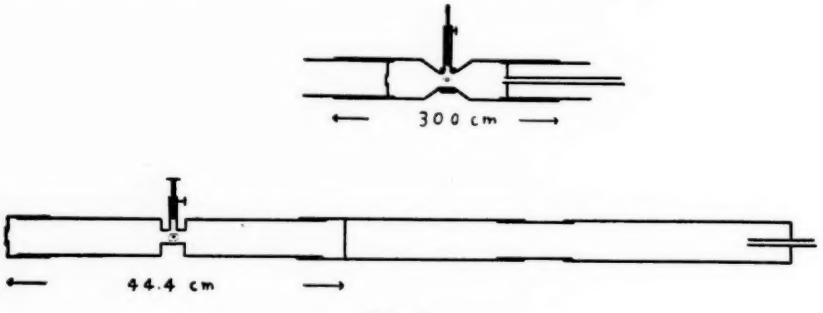

Fig. 1.

absence of other buildings, combined with the elevation of the site, made the reflection of sound nil, excepting that from the root itself. This reflection was diminished in two ways: first, by covering the adjacent portions of the roof with thick hair felt whose absorption coefficient is about 50 per cent. and by mounting the fork and horn approximately 150 cm. above the roof and separating them from each other not further than this same distance. Thus the effect of roof reflection was reduced to a small per cent. However, the proximity of the horn and fork

introduced an error in that the wave falling upon the horn was not strictly plane, though in all cases the wave surface deviated from a plane only by a small fraction of a wave-length.

Horns.—The horns were made of galvanized iron, U. S. standard plate No. 24. It was found that, if thin metal were used, there was resonance in the conical wall affording sufficient dissipation to affect the intensity in the horn itself. This effect could be demonstrated by changing the rigidity of the clamping of the horn. With the horns actually used, however, the intensity measurement was not capable of being changed by clamping and the walls did not vibrate noticeably. Inasmuch as the absorption of the metal when not vibrating is very small, it is assumed that the results here presented are those found in conical horns having rigid walls and possessing no absorption.

Each horn was cut off at the vertex, leaving a diameter of 0.5 cm.; thick wall rubber tubing 0.45 cm. in internal diameter and approximately 150 cm. in length, was slipped over the open vertex and connected to the Rayleigh disc.

In many of the experiments the lengths of the horns were altered a few centimeters in order to produce the length required for resonance. This was accomplished by means of heavy drawing paper slipped over the large end. It was experimentally shown that when the length of the addition of drawing paper did not exceed a few centimeters the paper served just as well as metal itself. The reason is, of course, that the changes in pressure near the large end of the horn are small.

Detailed Method.—Deflections of the disc were read and assumed (and justifiably so) to be proportional to the intensities in the horns. But in order that measurements of relative intensities with varying experimental conditions might be secured, corrections were made for the lack of constancy in the intensity of the source of sound, and for the change of sensitiveness of the disc caused by changes of temperature. The data for the first correction were obtained by microscopic measurement of the amplitude of the fork at the time of each reading of the disc. The second correction was secured through an occasional reading of the disc when attached to a tube terminating in a position fixed relative to the source of sound, the amplitude of the fork being noted. The details of applying these corrections are simple and almost obvious. The only assumption involved is that the intensity is proportional to the square of the amplitude of the fork.

II. EXPERIMENTAL RESULTS.

The experimental results herein presented are not as accurate as can be obtained, but they will give fairly satisfactory quantitative information concerning the action of conical horns. The experimental conditions

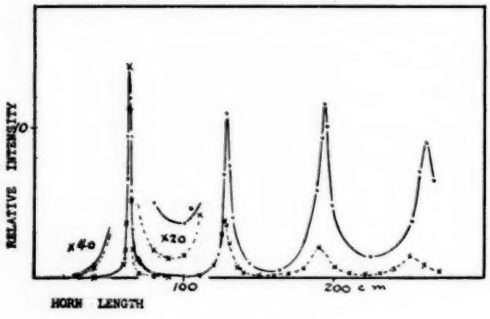

Fig. 2.

necessary to obtain more accurate results are difficult to secure. The experimental points shown in the curves are usually each the result of a single setting and observation. Nevertheless, they do not comprise the entire data but are merely typical. All results have been corrected for temperature, the reduction being made to 20° C.

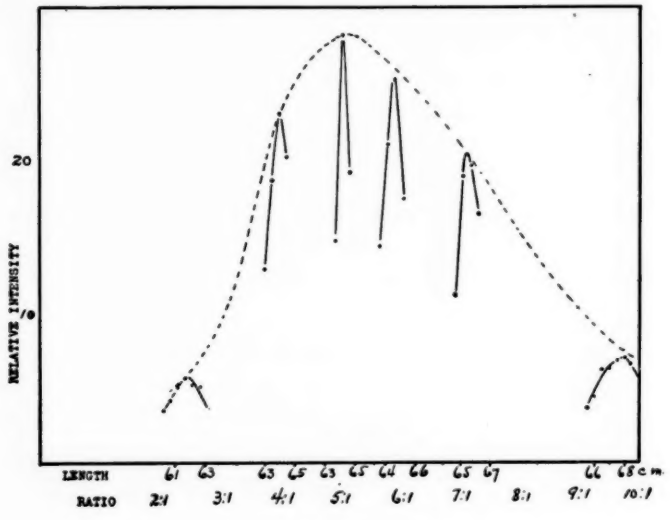

Fig. 3.

Variation of Intensity with Length, Frequency Constant.—The first question one might ask in regard to the action of a conical horn or tube acting as a receiver is in regard to the resonance characteristics and the

amplifying effect throughout the range of frequencies of interest. A series of trials was made with conical horns of constant angle but of varying length, using the same source of sound. The data were taken on several evenings and are shown, after corrections are made for the

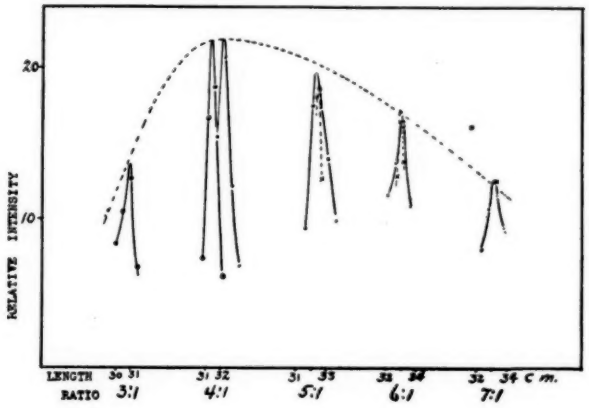

Fig. 4.

effect of temperature differences upon length, in the full line curve plotted in Fig. 2. Over two portions of the curve magnifications of 40 and 20 are also plotted. There should be added to the data in Fig. 2 an important relevant fact, namely, that the intensity without the horn as tested by an open tube at the position occupied by the open end of the

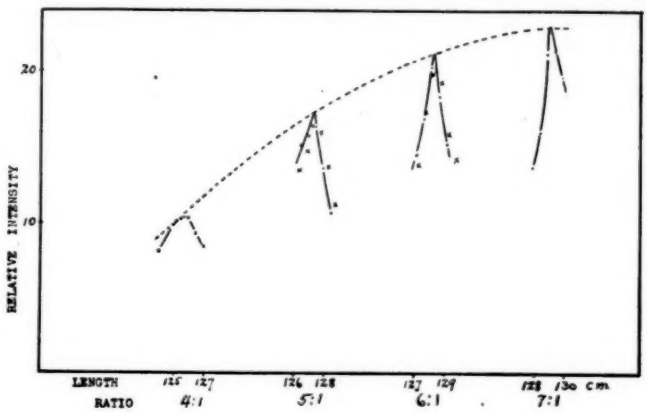

Fig. 5.

horn, was of the same order as the observation with the horn of shortest length.

Variation of Intensity with Angle in a Conical Horn, Assuming Maximum Resonance and Fundamental Frequency.—In Fig. 3 it is shown that

the maximum intensity at resonance occurs when the ratio of radial length to diameter of opening is approximately 5 : 1 for a frequency of 256 d.v.

Fig. 4 gives similar information in regard to the frequency of 512 d.v. Here the optimum ratio is approximately 4 : 1, that is, the radial length if four times the diameter of the open end. The observations indicated

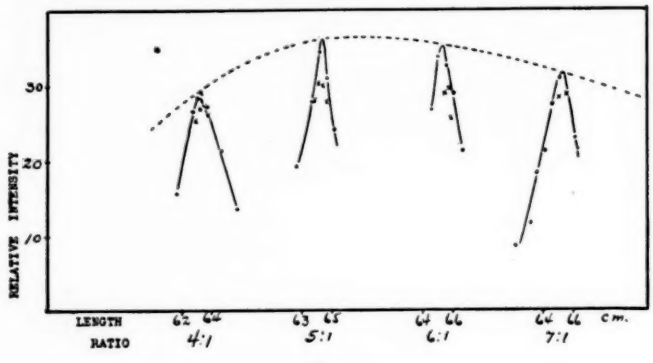

Fig. 6.

by circles and crosses were taken at other times. There appears a disagreement of peak location for the 4 : 1 probably due to an error in reading of 1 cm.

Resonance Peaks, Similar to those in Figs. 3 and 4, but with a Frequency of the First Overtone.—Fig. 5 for 256 d.v. and Fig. 6 for 512 d.v. are self-explanatory. Again crosses and dots indicate separate series. The point worthy of note is that, frequency constant, the optimum angle decreases when the horn is lengthened so that the fundamental becomes the first overtone.

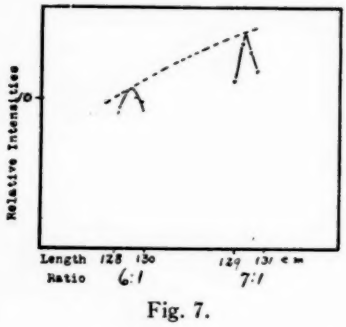

Fig. 7.

Resonance Peaks, with Frequency that of the Second Overtone.—With 512 d.v. only observations on two horns were taken and these are shown in Fig. 7. Although these data are too meager to interpret accurately, yet there is sufficient evidence that the optimum angle continues to decrease with an increase in the order of the overtone.

Variation in Optimum Angle with Order of Overtone.—Fig. 8 shows how the optimum angle changes with the order of the overtone and it is here seen that the ratio of change for the 256 and 512 is approximately the same. The reliability of the data gives only a general idea of the change.

Effect upon Resonance Characteristics of Conical and Cylindrical Attachments.—It is to be anticipated that the resonance characteristics of the tube attached to a conical horn for listening purposes will be superimposed upon the characteristics of the cone itself. By simple tests with a tonometer and attached cones and cylindrical tubes the writer found that the resonance frequencies of the attachments can be estimated by the following rule: The attached tube, cylindrical or conical, will introduce resonance characteristics which may be computed as to

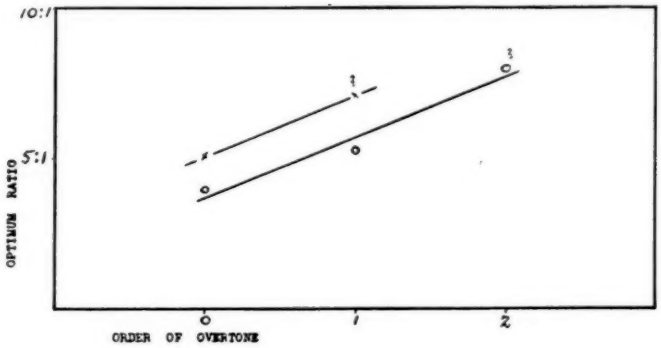

Fig. 8.

frequency by treating the end attached as a free open end but at the same time remembering that if the angle of the attached tube is less than that of the conical horn there is a positive end correction and if greater a negative end correction.

Directivity of a Conical Horn.—As is well known, when the source of sound is along the extended axis of the horn, the greatest intensity is produced at the vertex. This effect, because it depends upon the phase relations at the opening, becomes more marked the larger the opening of the horn. This comment is introduced merely because in practical application of the conical horn as a receiver, the exclusion of side noises is of great importance.

End Correction.—The experiments may be used to determine, the end correction of a conical horn. If we assume that,

$$L + KR = n\frac{\lambda}{2},$$

where L is the length of the horn, R the radius, K a correction factor, and n an integer, and plot L/n, and R/n as coördinates, K may be computed from the slope. If the results are in agreement with the above linear relation, it may reasonably be concluded that the resonance frequencies are integral multiples of the fundamental. The results of the experi-

ments have been so plotted and the assumption of the linear relation seems to be fully justified, whether the frequency is a fundamental or an overtone. The value of K appears to be 0.7, with a possible error of 10 per cent. Experiments can readily be designed to determine the exact relation between the resonance frequencies of the conical horn, and thus the end correction, with a much greater accuracy than here employed.

III. THEORY.

A brief reference to theory will aid in extending the application of the experimental results and in understanding the action of the conical horn. The only theory of which the writer is aware that offers a possibility of obtaining quantitative results for conical horns is that of Dr. A. G. Webster[1] of Clark University. In this theory he makes the following assumptions:

(*a*) That the tube or horn has a cross-section which is always small but is a function of the length, x.

(*b*) That the sound waves travelling in positive and negative directions of x are plane, *i.e.*, all variations in y and z directions are zero.

(*c*) That the conductivity for an open end is the same as a channel of a selected length.

(*d*) That the general equations of sound are applicable.

It will be necessary to extend the published work of Webster somewhat in order that we may secure the quantitative results. Let X be the volume of air periodically entering the horn under a maximum excess pressure p, and z be the impedance and equal to p/x (see Webster, loc. cit.). Let subscripts 1, 2, and 3 refer respectively to the values at the small end of the horn or in this case the vertex, the opening of the horn, and outside the opening at a point where the influence of the horn can be considered as vanishingly small. Assume the distance between p_2 and p_3 to be small compared with a wave-length and that Z_0 represents the impedance of the opening treated as a short tube or channel, the air escaping from a circular hole in an infinite plane. Then, by definition,

$$X_2 = \frac{p_2 - p_3}{Z_0} = \frac{Z_2 X_2 - p_3}{Z_0} = \frac{p_3}{Z_2 - Z_0}.$$

But according to Webster, equation (24)

$$X_2 = cp_1 + dX_1 = (cZ_1 + d)X_1,$$

where c and d are defined as follows:

$$c = \frac{-\sigma_2 x_1 \sin k(l + \epsilon_1 - \epsilon_2)}{\beta x_2 \sin k\epsilon_1 \sin k\epsilon_2}, \qquad d = \frac{\sigma_2 x_1 \sin k(l - \epsilon_2)}{\sigma_1 x_2}.$$

[1] Loc. cit.

σ_1 and σ_2 represent the cross sections at the small and large ends, respectively, $\beta = \rho a^2 k$, where ρ is density, a velocity of sound, and k the frequency divided by the velocity, x_1 and x_2 represent the distances from the vertex of the small and large ends respectively, and

$$l = x_2 - x_1,$$

$$\tan k\epsilon_1 = kx_1,$$

$$\tan k\epsilon_2 = kx_2;$$

$$\therefore X_1 = \frac{p_1}{Z_1} = \frac{X_2}{(cZ_1 + d)} = \frac{p_3}{(cZ_1 + d)(Z_2 - Z_0)}.$$

But since

$$Z_2 = \frac{aZ_1 + b}{cZ_1 + d} \quad \text{(Webster's equation (25))},$$

with

$$a = \frac{x_1}{x_2}\frac{\sin k(l + \epsilon_1)}{\sin k\epsilon_1}, \qquad b = \frac{\beta}{\sigma_1}\frac{x_1}{x_2}\sin kl,$$

then

$$p_1 = \frac{z_1 p_3}{(cz_1 + d)\left(\dfrac{az_1 + b}{cz_1 + d} - Z_0\right)},$$

and we have a relation between the exterior and vertex maximum excess pressures; and these pressures are proportional to the square roots of the respective intensities.

For the conical horn closed at the vertex,

$$z_1 = \infty,$$

and therefore

$$p_1 = \frac{p_3}{(a - cz_0)}.$$

Now Webster shows that, assuming the conductivity of the open end to be that of a short tube of conductivity c_0, ending in an infinite plane,

$$z_0 = \rho a^2 k^2 \left(\frac{k}{2\pi} i - \frac{1}{c_0}\right).$$

Hence the complete expression that will enable us to determine quantitative relationship between the incident sound intensity and that found at the vertex is

$$p_1 = \frac{p_3}{\dfrac{x_1}{x_2}\dfrac{\sin k(l + \epsilon_1)}{\sin k\epsilon_1} + \dfrac{\sigma_2 k x_1}{x_2}\dfrac{\sin k(l + \epsilon_1 - \epsilon_2)}{\sin k\epsilon_1 \sin k\epsilon_2}\left(\dfrac{k}{2\pi} i - \dfrac{1}{c_0}\right)}$$

and if p is to be determined for the vertex, $x_1 = 0$,

$$p_1 = \frac{p_3}{\dfrac{\sin kx_2}{kx_2} + \dfrac{\sigma_2 \sin k(x_2 - \epsilon_2)}{x_2 \sin k\epsilon_2}\left(\dfrac{k}{2\pi} i - \dfrac{1}{c_0}\right)}. \qquad (1)$$

It can be shown (Professor Webster has shown this, as well as the derivation just given by the author, but has not published either) that Webster's theory may be modified by omitting the assumption of small cross section, and by assuming spherical waves, p being a function of r, the radial length, and the time, with the following equation:

$$p_1 = \frac{p_3}{\frac{\sin kr}{kr} + \frac{\sigma \sin (kr - \epsilon)}{r \sin k\epsilon}\left(\frac{k}{2\pi} i - \frac{1}{c_0}\right)}, \qquad (2)$$

where

$$\tan k\epsilon = kr.$$

For our case of conical horns (2) is the more accurate.

For the conductivity c_0, it is assumed that the open end is equivalent to a channel if length 0.6 of the radius, Thus

$$c_0 = \frac{\pi R^2}{0.6 \times R} = \frac{\pi R}{0.6}.$$

With this value of c_0 and using (2), computations were made and relative values of $(p_1/p_3)^2$ or relative values of the intensity, were secured. In computing, one must of course use the modulus of the fraction in the above equation. The results are plotted in the dotted curve in Fig. 2.

An additional point of the theory should be noted. Equation (2) indicates that the computed values depend upon the product kr which is proportional to the product of radial length and frequency. In Fig. 2, the experimental results were plotted with changing radial length, frequency remaining constant. From the foregoing statement it is obvious that precisely the same curve would have been obtained with varying frequency, radial length constant, the values of the ordinates depending only upon the product of radial length and frequency.

IV. Discussion of Results.

Extension of Results. Fig. 2.—Inasmuch as we are apparently justified, by the preceding theoretical considerations, in regarding Fig. 2 as showing the relationship between relative intensity and frequency, the following conclusions may be stated:

(*a*) The frequencies producing resonance are very closely integral multiples of the fundamental frequency.

(*b*) The resonance peaks broaden with increasing frequency.

(*c*) The intensity values for the minima, located between the resonance peaks, increase with increasing frequency.

(*d*) The conical horn gives a very considerable amplification for any frequency nearly the same as or greater than the fundamental frequency.

(The ordinate at the beginning of the curve is in fact no greater than the relative intensity without the horn at all.)

(*e*) The greater the frequency, the less the difference between maxima and minima relative intensities, the curve promising to approach one of smaller and smaller undulations, yet one representing a great amplification.

These conclusions indicate that if a horn, long compared to the wave-length of the least frequency involved, is used as a receiver, it promises amplification without highly marked resonance characteristics. Experimental tests of this conclusion have been made, and with satisfactory verification.[1]

Utilization of Results.—In the utilization of the results of the foregoing experimental and theoretical discussion it must be remembered that the existence of the optimum angle and its alteration with the order of the overtone, prevents the experimental curve in Fig. 2 from being regarded as quantitatively typical. In fact, if the horn ratio had been approximately 7 : 1 instead of 5 : 1, the second resonance peak would have been higher than the first. In general, since there exists an optimum angle, the relative values of the amplification at the various resonance frequencies must depend upon the angle of the horn. It would be advantageous to have information concerning the effect of various frequencies, with a variation of the ratio and length of the horn, for then one could ascertain just what effects any conical horn would have, acting as a receiver for any sound of known frequencies. It is believed that the present results furnish a partial guide in the design of a conical horn for any purpose, particularly by way of suggestion as to what trials should be made to secure the desired horn.

Physical Aspects.—An explanation in physical terms will assist in understanding the causes for the characteristics of the curve in Fig. 2. Assume for the moment that the length of the conical horn is short compared with the wave-length, and that the horn opening is a part of a plane wall. The horn would thus reflect the sound with approximately the same effect as any other equal area of wall surface, or surfaces supposed to be perfect reflectors. We would then expect to find the horn acting as a reflector and not as a collector. This reflecting function of the conical horn persists irrespective of its dimensions, though of course as the length of horn becomes comparable to the wave-length of the sound, the reflection cannot be considered as in the same phase from the entire inner surface, but each element of area is a source of sound in such phase and amplitude that combined with the oncoming sound will result in a

[1] Stewart, PHYS. REV., XIV., 2, 1919, p. 166.

zero value of the velocity potential over the wall surface. In the conical horn, then, reflection takes place throughout its entire length, this reflection being in all directions away from the inner surface just as if each element of that surface is a source of sound.

It is a consideration of this reflection which makes clear the reason for the lack of concentrating ability shown in that part of Fig. 2 to the left of the first resonance peak. Even when the radial length of the horn is approximately one-half of that for maximum resonance, the intensification produced by the horn is almost unnoticeable. In other words, the horn acts much like a plane reflector that subtends the same angle at the source and the energy is approximately the same at the vertex as if the opening were a hole in that plane.

Just as there is reflection from the walls of the incoming wave, so also there is reflection of the outgoing wave. In this latter reflection from the walls we find an explanation of the fact that the minima in Fig. 2 are not constant, showing zero amplification. Conditions for resonance demand reflection not only at a distant end but also at the opening where the incoming wave enters. In the conical tube, inasmuch as we have reflection of the outgoing wave throughout the entire length of the wall, there is always a certain amount of reflection at the point where the phase of the incoming wave and the just reflected portion of the outgoing wave are the same. There would thus be "resonance." According to this explanation there should be resonance for any frequency greater than the fundamental, for with every such frequency there is in at least one region of the inner surface a reflection of the outgoing wave with the above stated phase relations, thus furnishing the necessary conditions for resonance. Fig. 2 is in agreement with this conclusion in that the minima show a distinct amplification. As an illustration of the above explanation, suppose the horn has a length corresponding to the first minimum of Fig. 2. The reflection from the opening of the outgoing wave is opposite (approximately) in phase to the incoming wave. In a cylindrical tube this would be the cause of zero amplification. But in the conical horn there is reflection from the wall near the opening which, being reflection without change of phase, will be in phase (approximately) with the incoming wave. Resonance will therefore exist.

In this connection the writer wishes to emphasize that the usual view of the conical horn, *i.e.*, that it acts primarily as a concentrator, is misleading, and that the view of it as a resonator will, *in most applications be much more satisfactory and helpful.* Of course, both concentration and resonance are present and which predominates depends upon the dimensions of the horn and of the length of the wave.

In the usual theory, giving the resonating frequencies in conical tubes,[1] it is assumed that the incoming and outgoing waves are spherical within the tube and the results derived for the natural frequencies are approximately correct. But the assumption of sphericity is in effect an assumption of reflection from the walls throughout the length for otherwise there could not exist these imagined portions of spherical waves. In a portion of Webster's theory, to which reference has been made, plane waves are assumed in the case of a tube whose cross-section is always small, and spherical waves in the tubes not so limited, but there are applied hydrodynamic equations which permit reflections to exist. In the assumption of an approximately plane wave or a spherical wave reflections are virtually assumed, for these are necessary to maintain the stated shape of the wave. Thus it will be seen that existing theroies of the action of conical horns give approximate results for the natural frequencies because they distinctly assume reflection throughout the length of the walls, though without specific mention.

There are further points which should be considered from a phenomenon viewpoint, especially since our theory gives us no direct assistance toward an explanation. Two of these are the reasons for an optimum angle and the variation of the optimum with frequency and with the order of overtone. It is believed by the writer that the existence of the optimum angle is occasioned by the fact that the horn has a diameter not vanishingly small as compared with a wave-length and hence the vibrations are not merely radial. Obviously then, certain angles would be expected to give the optimum resonance. A theoretical investigation of these nonradial vibrations is now being made and a further discussion of the optimum angle will be postponed.

Errors.—The experimental results exhibited in this report should be interpreted in the light of the conditions under which they were undertaken. The results are and must be inaccurate because the conditions that would make possible accurate measurements cannot be had. In these experiments the horn is really a frustrum of a cone with a flux of energy through the "listening" tube attached at the several vertices. Inasmuch as the effect of the tube upon the intensity at the point of attachment in any case depends upon the area, length, damping, and resonance characteristics of the tube, it is not possible to establish experimentally a universally applicable statement of variation of relative intensification of the sound as dependent upon the length and angle of the horn and frequency of the sound used. It is to be observed, however, that, with the area and the length of the rubber tubing used in these

[1] See Rayleigh and also Barton, loc. cit.

experiments, the flux of energy would seem not to be great enough to destroy the value of the relative intensities obtained regarded as an approximation to the truth, that the damping in the tubing was sufficiently great to make the resonance in itself very small indeed, and that, in fact, inasmuch as the length of the tubing was kept constant and different frequencies are not compared, the small resonance effect which might exist did not seriously modify the variation of relative intensity with the angle and length of the horn. It is believed by the writer that, when all the physical conditions are given appropriate consideration, one is justified in accepting the results herein described as furnishing a fair idea of the relative intensities at the vertices of conical horns.

In conclusion, I wish to acknowledge the valuable assistance of Mr. Eugene M. Berry.

PHYSICAL LABORATORY,
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THE HIGH FREQUENCY SPECTRA OF LEAD ISOTOPES.

BY C. D. COOKSEY AND D. COOKSEY.

SYNOPSIS.

Least Detectable Difference in Wave-length Dependent on Distance between Lines and Length of Reflected Ray.—Reasons are given for confining the work to a comparison of the $L\alpha_1$ lines of ordinary and uranio-lead. The least detectable difference in wave-length between two lines on the same plate is directly proportional to the least detectable distance between their axes and inversely to the length of the reflected ray between the crystal and plate. However, the nature of the $L\alpha_1$ line of lead is such as to make it more important to reduce as much as possible the least detectable distance between the lines, than it is to increase the length of the reflected ray.

Limit of Accuracy. Special Method of Suspending and Displacing Plateholder.—The spectrometer is described in detail in the preceding article. A special method of suspending the plateholder and giving it a known displacement is described, by which it is possible to fix the limit of accuracy directly. An X-ray bulb of the gas type was used with a specially designed anode which could be rotated so as to bring different parts of its face under the action of the cathode stream. The two kinds of lead were placed on different parts of the same anode. The uranio-lead was from a primary uraninite from India.

Upper Half of Line from One Lead and Lower Half from Other Photographed on Same Plate.—The α_1 line of one kind of lead was photographed on the upper half of the plate, and the same line from the other lead on the lower half, immediately below it. Full lines for reference were sometimes photographed near the half lines for measurement purposes. The limit of accuracy was found by giving the plate a known displacement between the taking of one half and the other, of a line from the same kind of lead.

$L\alpha_1$ Lines from Ordinary and Uranio-lead Do Not Differ in Wave-length by as much as 0.005 Per Cent.—It is found that a displacement between two half lines of $(10)^{-3}$ mm. can be readily detected and that the wave-lengths of the $L\alpha_1$ lines of the two kinds of lead can not differ by as much as $0.6 \times (10)^{-4}$ Å., or 0.005 per cent.

INTRODUCTION.

A KNOWLEDGE of the relation between the spectrum of a substance and that of its isotope is important in that it may throw further light on the structure of the atom. Some work in this line has been done. Aronberg,[1] working with a grating spectrograph, has reached the conclusion that the wave-length of the line λ 4058 is greater by 0.0043 Å. for lead of radioactive origin than it is for ordinary lead. The work of Aronberg has recently been corroborated by Merton,[2] working with a

[1] Aronberg, Astrophysical Jour., Vol. 47, p. 96, 1918.

[2] Merton, Proc. Roy. Soc., No. A, 679, Vol. 96, p. 388, 1920.

Fabry and Perot *étalon*. The X-ray spectra of lead isotopes have been investigated by Duane and Shimizu,[1] and by Siegbahn and Stenström.[2] The former authors, using an ionization method, measured the three critical absorption wave-lengths in the L-series from a specimen of ordinary lead and one of radio-active lead, the atomic weights of which had been determined by Richards and found to differ by more than ½ per cent. They conclude that these wave-lengths do not differ between the two kinds of lead by more than 0.1 per cent. or about 0.0008 Å. Siegbahn and Stenström used a photographic method and investigated all the lines in the L-series, and the strongest lines in the M-series of ordinary and radioactive lead. The spectra of the two kinds of lead were photographed on the same plate under the same conditions. By using a lead screen to cover one part or another of the plate at a time, the middle portion of the lines of one kind of lead were obtained with the upper and lower portions of the lines of the other kind of lead above and below. The authors conclude that the wave-lengths of the two kinds of lead do not differ by more than 0.0005 Å. They do not, however, in the article referred to, state how they estimated their limit of accuracy. The dispersion and width of slit used are not mentioned. For a description of their apparatus they refer to a number of the Jahrbuch der Radioaktivitat und Elektronik for 1916 which, up to the present, has not come to hand on account of the war.

Some time ago Professor Uhler suggested to the senior author that it would be of interest to investigate the X-ray spectra of isotopes, and as we had a supply of uranio-lead at our disposal, it was decided to undertake the work. On account of delays caused by the war we did not receive notice of the work of Siegbahn and Stenström until our work was well under way. The question of the spectra of isotopes seemed, however, of sufficient importance to warrant a repetition of the work by independent investigators even though the methods used by each were nearly the same. At the same time we had reason to believe that we could attain a sufficient degree of accuracy to detect a difference of wave-length, if present, well within the limits set by the authors named. We therefore decided to complete the work.

The potential required to produce the K-series of lead is so high that it is doubtful if this series can be produced in the ordinary gas type tube. On the other hand the wave-lengths of the L-series are long enough to make the absorption of the air an appreciable factor. Not having a vacuum spectrometer at our disposal we decided to confine our investigation to the α_1 line of the L-series of lead.

[1] Duane and Shimizu, Proc. Nat. Acad. Sci., No. 6, Vol. 5, p. 198, 1919.

[2] Siegbahn and Stenström, Phys. Zeitschr., Vol. 18, p. 547, 1917.

The $L\alpha_1$ line of lead is very broad and more resembles a band than a line. This is probably a characteristic of the lines of the L-series in general. A line of this nature will always be broader than the slit even when the plate is placed exactly at the focus of the slit, and the higher the dispersion, the greater will be this difference. The smallest difference in position between two such lines, taken in a manner similar to that described by Siegbahn and Stenström,[1] which can be detected will depend on the narrowness, sharpness, and intensity of the lines. Since these fall off as the dispersion is increased there is nothing to be gained by carrying the dispersion beyond a certain point.

If a photographic plate is so placed on the spectrometer that its plane is normal to the collimating line, and at such a distance from the axis of rotation of the spectrometer that the portion of the plate that is to receive the spectral line is either at the focus of a single slit, or somewhere between the foci of two slits of equal width, then it is easily shown that the distance, δ, between the axes of two lines whose difference in wave-length, $\Delta\lambda$, is small, is given by:

$$\delta = \frac{r\Delta\lambda}{d \cos\theta \cos 2\theta},$$

where r is the distance from the axis of rotation to the plate, measured along the reflected ray, d is the grating space of the crystal, and θ is the glancing angle. For the $L\alpha_1$ line of lead and the (100) planes of calcite, θ is approximately 11.°18 and d is 3.03 Å., which gives

$$\Delta\lambda = 2.8 \text{ Å.} \left(\frac{\delta}{r}\right). \qquad (1)$$

The smallest difference in wave-length that can be detected is therefore directly proportional to the smallest detectable value of δ, and inversely proportional to r. Stated in this way however, and for the line under consideration, δ is some direct function of r, and it becomes more important to reduce δ to a minimum than to increase r.

APPARATUS.

We used a spectrometer, a detailed description of which will be found in the preceding article. The plateholder was always mounted in such a manner that its plane was approximately normal to the line of collimation and at such a distance from the axis of rotation that the part of the plate where the spectral lines were produced was always either at the focus of one or other of the slits S_1 and S_2, or between these foci.

For the special purpose of this experiment we mounted a screen

[1] *Loc. cit.*

directly in front of the plateholder, but not touching it. The screen consisted of a horizontal strip of brass about 1.7 mm. thick, of about the same length, and a little more than half the breadth of the opening in the front of the plateholder. The screen was supported by uprights, which stood on the same masonry pier with the spectrometer, but which were otherwise entirely independent of the latter. It could be raised entirely above the opening in the plateholder, or lowered so as to cover either the upper or lower half of this opening as desired.

For a purpose to be explained later, the plate holder, instead of being supported on the azimuth circle clamped to the T rail, could be hung on two parallel wires of equal length, one fastened at each end of the top of the plateholder. The wires were about 134 cms. long, and their upper ends were fastened to a beam above the spectrometer. When the plateholder was supported in this way its regular mounting was removed from the T rail. In place of this the plateholder had screwed to its base a brass plate with two vertical posts extending below it and about 10 cm. apart. Through the lower end of each post and normal to the plane of the plate extended a screw with a smooth rounded point. By suitably adjusting the supporting wires, a small component of the weight of the plateholder served to make these screws bear gently against a smooth glass plate clamped to the T rail. The frame of a spherometer was clamped to one end of the T rail so that the axis of the screw was parallel to the long axis of the photographic plate, and the point of the screw bore against a smooth glass plate fastened on the end of the plate holder. The spherometer screw had a pitch of 0.5 mm. and its circular scale had a least count of 0.001 mm. To facilitate turning the screw accurately through one division of the head without rocking it in its nut, a tangent screw was provided.

For diffraction grating we used the cleavage planes of a specimen of calcite crystal which had already been found to give very satisfactory lines.

The X-ray bulbs were of the gas type and were made especially for us by Mr. A. Greiner, vice-president of the Green and Bauer Company, Hartford, Conn.; and the skillful manner in which he followed our design greatly facilitated the experimental manipulation. The one from which we obtained the fianl results had a water-cooled anode which could be rotated or removed by means of a ground glass conical joint. The anode was a copper disc with a silver disc about 2 mm. thick screwed to its face. The plane of the anode was made as nearly normal as possible to the axis of revolution of the ground glass cone. The anode was so mounted that its face made an angle of about 45° with the cathode rays,

and so that different parts of its face came under the action of these rays when the ground joint was rotated. A short side tube was sealed to the main bulb so that the axis of the tube was normal to the plane containing the cathode rays and the axis of the ground joint. The end of this side tube was covered by a brass plate with a small hole in it. The hole, in turn, was covered with aluminum foil about 0.12 mm. thick. The line through this hole to the focal spot made a small angle with the plane of the anode.

The X-ray bulb was placed with the aluminium window close to the slit, S_1, of the spectrometer, with the plane of the anode vertical, and so that the line from the focal spot through the aluminium window should be as nearly horizontal as possible. It was supported on three very rigid supports clamped to the anode tube, cathode tube, and an extra side tube respectively. These supports were fastened to the same table on which the spectrometer rested. This table consisted of a heavy granite slab cemented on a brick pillar isolated from the building.

The specimens of lead to be compared were both placed on the anode at the same time in the following manner. The position of the focal spot on the anode was marked by running the bulb; the ground joint was then turned through about 45°, and the new position of the focal spot marked. The positions of the ground joint corresponding to these two positions of the anode were marked on the outer end of the cone. The anode was then removed from the bulb, and the silver disc taken off. A cluster of small holes, as close together as possible, was drilled in the silver at both positions of the focal spot. The holes were about 1 mm. in diameter, and enough were drilled to more than cover the focal spot. One of the kinds of lead to be investigated was pressed into one cluster of holes, and the other kind into the other cluster. Great care was taken not to contaminate either lead with the other.

The pressure within the X-ray bulb, while running, was maintained as nearly constant as possible by a mercury diffusion pump so that the bulb would back a 12.7 cm. parallel spark gap. The current through the bulb was about 5 milliamperes, supplied by an "Ideal Interrupterless" X-ray current generator, purchased from the Kny-Scheerer Company of New York City.

The specimen of ordinary lead was cut from ordinary commercial sheet lead taken from the stock in the laboratory. The uranio-lead was very kindly prepared and furnished by Professor B. B. Boltwood with the statement that it was obtained from some lead chloride that had been separated from a primary uraninite from India; that it should be entirely free from ordinary lead, and should contain only a little "thorio-lead," if such a substance exists.

METHOD OF COMPARING SPECTRA.

The method employed to compare the spectra of the two leads was as follows. The plateholder was placed on its usual mounting on the T rail, and one half of its breadth covered by the brass screen. It was then exposed to the rays from one kind of lead, the crystal always being given a small rotation back and forth around the position which gave the maximum reflection. At the end of the exposure the anti-cathode was turned until the other kind of lead came under the focal spot, and the plate exposed again with its other half covered by the screen. On some plates a full line was taken on each side of the half lines and about 2 mm. from them. This was to give fiducial lines from which to measure. Plates were taken at various distances from the axis of rotation of the spectrometer and with various distances between this axis and the slit S_1. The distance from the axis to slit S_2 was always 4.5 cms. Different widths of slit were tried, and sometimes one, and other times two slits were used.

The plane of the anode did not remain absolutely fixed when the ground joint was turned to bring one or the other lead under the focal spot, though the variation was very slight. However, when using two very narrow slits a long distance apart, it was necessary to realign the spectrometer after turning the anode. This could easily be done by rotating the base of the spectrometer about the long axis of S_1 as described in the previous article. The spectrometer was lined up for one kind of lead and the position of the base noted. The other kind of lead was then brought into position, and the spectrometer realign for it. The first half line was then taken without changing the anode. Then, after the first kind of lead had been brought back into position, an auxiliary plate was introduced close to S_2 and a series of exposures of equal length taken on it for different recorded positions of the spectrometer base around the position which had been noted. The position which gave the most intense image of S_2 was then used in taking the second half ine. Absolutely no part of the spectrometer, slits, crystal or plateholder, were ever touched from the time the exposure for the first half line was started till the exposure for the second half was completed, except to rotate the base of the spectrometer to adjust the line up. This was done by slow motion screws, the base resting at two points on *plane* surfaces and pivoted at a third point on a carefully made cone. The auxiliary plate was supported without touching any part of the spectrometer.

TEST FOR LIMIT OF ACCURACY.

We wished, if possible, to compare the spectra of the two kinds of lead to a much higher degree of precision than had previously been attained. We were therefore confronted with the problem of finding what was the smallest displacement that we could detect between two half lines taken in the manner described. To do this we proposed to give the photographic plate a known displacement after taking one half line and before taking the other, both halves being produced by the same kind of lead. Various means of supporting and displacing the plateholder were tried for this purpose and proved unsatisfactory. Finally, at the suggestion of Professor Uhler, we hung the plateholder on wires in the manner already described and displaced it with the spherometer screw.

RESULTS.

1. *Limit of Accuracy.*—Two plates, nos. 41 and 44, were taken with the plateholder suspended on the wires. Slit S_1 was wide open, and S_2 was 0.02 mm. wide. The value of r, equation (1), was 4.5 cms. The crystal was rotated back and forth through an angle of $1\frac{1}{2}'$ on each side of the best setting. On No. 41 five exposures were taken using the uranio-lead only. The first exposure was a full line, the third and fourth were upper halves, and the second and fifth were lower halves. The plateholder was displaced by the screw in the same direction after each exposure; 2.000 mm. after the first, 0.002 mm. after the second, 2.000 mm. after the third, and 0.004 mm. after the fourth. On No. 44 only four exposures were taken, using the ordinary lead. The first and fourth were full lines, the second was a lower, and the third an upper half. The displacements after the first and third were both 2.000 mm., and that after the second was 0.001 mm.

Both the small displacements between adjacent half lines on plate No. 41 were quite obvious under a low power magnifying glass. The displacement of 0.001 mm. on No. 42 was not visible in this manner. The lines on both plates were sharp and black and not more than 0.025 mm. broad. Both plates were measured on a measuring engine, frequently tested and regularly used by Uhler and others for measuring plates obtained with the large Rowland spectrograph. The measurements were made along two lines normal to the spectral lines, one a small distance above the break between the halves and the other the same distance below. The measured values of the distances agreed very closely with the actual displacements. The values of the small displacements, obtained by subtracting the upper from the lower distances, in no case differed from the actual displacements by as much as 0.001 mm. We

therefore feel justified in assuming that with lines of the sort obtained on these two plates, a displacement of 0.001 mm. could not escape detection.

2. *Comparison of Leads.*—The following plates were taken with two exposures, giving the upper half of the line from one lead and the lower half from the other, on each, the plateholder remaining fixed during the two exposures, and the crystal rotated back and forth around the best setting.

Plate No. 6.—The width of S_1 was 0.04 mm., and S_2 was wide open. The distance from S_1 to the axis of rotation was 35 cms. and the value of r was 35 cms. The ordinary lead line was above, and the uranio-lead line below. The lines were very fuzzy, though dark, and about 0.1 mm. broad.

Plate No. 7.—The data for this plate are the same as for No. 6 except that S_1 was closed to 0.02 mm. and the positions of the lines from the two leads were reversed. The lines were not quite as broad as on plate No. 6, but were weaker and just as fuzzy.

Plates No. 29 *and No.* 30.—The width of both S_1 and S_2 was 0.02 mm. The distance of S_1 to the axis of rotation was 20 cms. and the value of r was 20 cms. The positions of the lines were reversed on the two plates. The lines were narrower than on the other two plates and were fairly strong, but were still too broad and fuzzy to permit of precise measurements.

None of these plates showed any displacement between the lines, but their axes were too indefinite to permit of their being compared with a high degree of precision. An attempt was made to set a superior limit to the displacement that could exist without being visible. These plates were placed on the measuring engine, and one cross hair of the microscope was set as nearly as possible parallel to, and on what appeared to be the axis of the pair of half lines considered as one continuous line. The engine screw was then turned the least amount that would throw the cross hair, beyond question, off the axis of the lines. In this manner it was estimated that, on the first two plates, there could not have been a displacement as great as 0.02 mm., and on the second two, as great as 0.01 mm. If these values are assigned to δ in equation (1), with the corresponding values of r, it is seen that the possible difference of wave-length between the lines of the two kinds of lead can not exceed $1.6\ (10)^{-4}$ Å. on the first two plates, and one half this value on the second two.

In order to have lines which could be measured with as high a degree of accuracy as those obtained in the tests for the smallest detectable displacement, we took two more plates, No. 46 and 47. The conditions

under which these plates were taken were as follows. Both slits were 0.02 mm. wide. The distances from S_1 and S_2 to the axis of rotation were 15.6 cms. and 4.5 cms. respectively. The value of r was 4.9 cms. The crystal was rotated back and forth through an angle of $1\frac{1}{2}'$ on each side of the best setting. Four exposures were taken on each plate, namely: a full line A was taken for one kind of lead, the plateholder was displaced along the T rail about 2 mm., and the lower half, B, of the line from the same kind of lead was taken; the plate holder remaining fixed, the upper half, C, of the line from the other kind of lead was taken, and then the plateholder was displaced about 2 mm. further and a full line, D, from the same lead as C, was taken. The direction of the displacement of the plateholder was such as to place A on the long wave-length side of B and C, and D on the short.

TABLE I.

Plate No. 46.

Upper Half.			Lower Half.		
(*AC*).	(*AD*).	(*CD*).	(*AB*).	(*AD*).	(*BD*).
2.1355	4.2021	2.0666	2.1384	4.2038	2.0654
.1341	.2033	.0692	.1373	.2051	.0678
.1370	.2038	.0668	.1356	.2035	.0679
.1358	.2059	.0701	.1363	.2061	.0698
.1363	.2043	.0680	.1357	.2048	.0691
Mean 2.1357	4.2039	2.0681	2.1367	4.2047	2.0680

$(AB) - (AC) = 0.0010$
$(CD) - (BD) = 0.0001$
Mean $= 0.0006$

Plate No. 47.

Upper Half.			Lower Half.		
(*AC*).	(*AD*).	(*CD*).	(*AB*).	(*AD*).	(*BD*).
2.0020	4.0512	2.0492	2.0045	4.0534	2.0489
.0045	.0522	.0477	.0075	.0550	.0475
.0044	.0522	.0478	.0021	.0538	.0517
.0064	.0550	.0486	.0046	.0539	.0493
.0036	.0524	.0488	.0057	.0533	.0476
.0031	0.517	.0486			
Mean 2.0040	4.0525	2.0485	2.0049	4.0539	2.0490

$(AB) - (AC) = 0.0009$
$(CD) - (BD) = -0.0005$
Mean $= 0.0002$

On plate No. 46 the lines A and B were produced by uranio-lead and the lines C and D by ordinary lead. On plate No. 47, on the other hand, the lines A and B were produced by ordinary lead, and C and D by uranio-lead. The lines on both plates were well defined and sharp though not quite as black as on plates 41 and 44. The upper and lower halves of each plate were measured on the engine. These measurements are set forth in detail in Table I., the figures in each column being the various measurements of the distance between the two lines corresponding to the letters at the head of the column. The units are approximately millimeters.

It will be seen from the table that on plate No. 46 the measurements from both A and D assign to C, an ordinary lead line, a longer wave-length than B, a uranio-lead line. The measurements from A on plate No. 47, however, assign to C, a uranio-lead line, a longer wave-length than B, an ordinary lead line, while the measurements from D reverse this order. None of the displacements between the half lines deduced from the measurements of these plates are greater than 0.001 mm., and the means are much less. Obviously the conclusion to be drawn from these two plates is that there is no difference in wave-length between the two kinds of lead great enough to produce a shift in the line of as much as 0.001 mm. With the value of r used with these plates we feel justified in the conclusion that the difference in wave-length between ordinary and uranio lead for the α_1 line of the L-series can not be as great as 0.00006 Å. The wave-length of this line being about 1.18 Å., the difference, if it exists at all, must be less than 0.005 per cent.

In conclusion it gives us great pleasure to express our sense of obligation to Professor Uhler for his interest and many valuable suggestions in connection with the work, and to Professor Boltwood for supplying us with the specimen of uranio-lead used.

SLOANE PHYSICS LABORATORY,
YALE UNIVERSITY,
March 21, 1920.

ON THE VARIATION OF THE FACTOR h IN THE EQUATION $\frac{1}{2}mv^2 = h\nu$.

BY FERNANDO SANFORD.

SYNOPSIS.

Relation between the β- and γ-rays of Ra B; Theory.—The author has previously shown that by combining the equations for orbital motion of an electron about a central positive charge with the equation $\frac{1}{2}mv^2 = h\nu$, the magnitude of the central positive charge about which a radiating electron is revolving may be calculated, and that when this value of the central charge is equated with one derived from an equation of the form of Moseley's equation for the frequency of X-rays the wave-length of the shortest lines in the K-radiation of elements of atomic numbers from 35 to 58 may be calculated as accurately as they can be measured. Later measurements of wave-lengths of elements with higher atomic numbers show that the computed wave-lengt hs vary progressively from the measured wave-lengths as the atomic numbers increase. It is also known that Moseley's law for the frequency of X-rays varies in the same manner.

The equation from which the author calculates the wave-length is

$$\frac{2^{1/2}h^{3/2}c^{1/2}}{m^{1/2}\lambda^{1/2}e\pi} = 2e(N - 3.6).$$

Since the right-hand member of this equation shows that the nuclear charge in the case of K-radiation increases by $2e$ in passing from one element to the one of next higher atomic number, it is not probable that this law breaks down at atomic number 58, and that the nuclear charges of elements of higher atomic number increase by variable quantities. The failure of the equation to apply to elements of higher atomic number must then be looked for in the left-hand member. In this member the only variable quantities (since λ is measured) are m, the mass of the electron, and possibly h, the Einstein factor. These quantities appear in the form of the ratio h^3/m, and the author has calculated values of h^3/m which substituted in his equation will give the correct wave-lengths for elements of higher atomic number. These values of h^3/m are then plotted against their corresponding wave-lengths and a smooth curve is obtained. Values of h^3/m are then taken from this curve for a number of the γ-ray wave-lengths measured by Rutherford and Andrade. These wave-lengths are then assigned to certain groups of β-rays from Radium B on grounds which seem to the author permissible. The mass, m, and the quantity h corresponding to the speeds of these β particles were then computed. The values of h computed in this way seem to be derivable from m by the equation $h = 4.25 \log m + 2.50 \cdot 10^{-27}$. Using this value of h, wave-lengths were computed for the fourteen groups of β-rays from Radium B whose speeds were measured by Rutherford, and nine of them were found to agree satisfactorily with wave-lengths in the γ-radiation of Radium B and Radium C, while two more of the measured wave-lengths seem to correspond with the speeds of β-particles from Radium C.

Calculation of Orbital Velocities of Radiating Electrons by Substituting 4.25 log m + 2.50·10⁻²⁷ for h in the equation $\frac{1}{2}mv^2 = h\nu$.

Using a series of values of v from $0.1 \cdot 10^{10}$ to $2.2 \cdot 10^{10}$ corresponding values of

m were calculated from Bucherer's equation for the transverse mass of an electron, viz., $m = m_0/\sqrt{1 - v^2/c^2}$. From these values ν was computed using the equation $\nu = \frac{1}{2}mv^2/(4.25 \log m + 2.50)$, and these values of ν were plotted on a large scale as abscissas using the values of v from which they were derived as ordinates. From this curve were taken values of v corresponding to all the highest frequencies of K-radiation as measured by Siegbahn and his colleagues and Duane and his colleagues. The values of v obtained in this way are then compared with values calculated from the equation $v = 2.054(N - 1.66)$ and are found to agree satisfactorily with no tendency to vary progressively with the higher numbers, as do the values calculated by Duane using h as a constant.

It is concluded that since h may be replaced in the equation $\frac{1}{2}mv^2 = h\nu$ by a function of m with a great improvement in the accuracy of the equation, there is no longer ground for speaking of an energy quantum in this equation.

THE writer has shown elsewhere[1] that by combining the equation for orbital motion of an electron about a central positive charge, viz., $mv^2/R = Qe/R^2$, with the equation $\frac{1}{2}mv^2 = h\nu$ it is possible to compute the central positive charge, Q, of a radiating atom, and from this value of Q and a corresponding value computed by an equation of the form given by Moseley to determine the wave-lengths of all known characteristic X-ray bands. Also, by combining the equation for Q with the formulæ for computing the wave-length of the lines in the various spectral series it was shown that the unit electrical charge may be computed from any line in any known spectral series of hydrogen or helium in terms of the wave-length and the serial number of the line. It is also possible by the use of the same equation to calculate the value of the Einstein factor h from the convergence charge of the Balmer series in hydrogen.

The above mentioned computations are not based upon any theory of atomic structure except that radiating electrons are in orbital revolution about a central positive charge, and that the frequency of revolution of the electron determines the wave-frequency of its radiation. In the opinion of the writer these assumptions are not incompatible with any of the commoner theories of atomic structure, but are admittedly in conflict with the Bohr theory of radiation.

The expression for the value of the central charge, Q, derived from the above equations is

$$Q = \frac{2^{1/2}h^{3/2}c^{1/2}}{m^{1/2}\lambda^{1/2}e\pi}.$$

By equating this value of Q with a value derived from an equation of the form of the Moseley equation we may get an expression for the

[1] See especially On The Nuclear Charges of Atoms, PHYSICAL REVIEW, IX., 383 (1917); The Astronomical Atom and the Spectral Series of Hydrogen, Astrophysical Journal, XLVIII., 1 (1910); The Helium Spectrum and the Unit Electrical Charge, Astrophysical Journal, XLIX., 337 (1919).

value of λ. Thus for the characteristic K-radiation $Q = 2e(N - 3.6)$ where N is the atomic number. Then for this radiation

$$\frac{2^{1/2}h^{3/2}c^{1/2}}{m^{1/2}\lambda^{1/2}e\pi} = 2e(N - 3.6).$$

This equation has been found to apply to the elements from $N = 35$ to $N = 58$ so that the wave-lengths of the K-radiation from these elements may be calculated from this formula as accurately as they may be measured.[1]

In the PHYSICAL REVIEW of January, 1920, the present writer undertook to show that if the speed of the β-particles sent off from Ra-B and Ra-C is related by Einstein's law to the accompanying γ-radiation then before the expression $\frac{1}{2}mv^2$ may be substituted for Ve in that equation the factor h, as well as m, must be assumed to vary with the speed of the β-particles.

Since the article referred to was written, two papers which have an important bearing on this question have appeared. In the Physikalische Zeitschrift of June 1, 1919, Siegbahn and Jönssen have published new determinations of the wave-lengths of highest frequency in the K-radiation series of a considerable number of elements from cadmium to uranium, and in the Physical Review of December, 1919, Duane and Hang-Fuh-Huh and Duane and Shimizu have extended the wave-length measurements on the absorption frequencies of the K-radiation to elements of both lower and higher atomic numbers than those whose frequencies were determined by Blake and Duane. These measurements when taken with the wave-length determinations of Siegbahn and Stenström in 1916 give a series of measurements of the short wave-lengths in K-radiation for most of the elements from atomic numbers $N = 12$ to $N = 92$.

When these newly determined wave-lengths are computed by our formula it is found that the correspondence is equally as good as for the measurements of Blake and Duane up to an atomic number 58, but that from this point onward the calculated values are greater than the measured values. Siegbahn and Jönssen call attention to the fact that from this point onward their wave-lengths do not follow Moseley's law, and they attribute the variation from this law to the variable mass of the electron at high speeds.

In the papers in the PHYSICAL REVIEW, Duane and his colleagues calculate a value of v for each of the elements whose wave-lengths they have measured by using the equation $\frac{1}{2}mv^2 = h\nu$ and computing the value of m at each speed by the Bucherer equation, $m = m_0/\sqrt{1 - (v^2/c^2)}$.

[1] See PHYSICAL REVIEW, XV., 68 (January, 1920).

They found that their values of v computed in this way bore a straight line relation to the atomic numbers, as does the square root of the frequencies in Moseley's equation, but that for atomic numbers above 58 their calculated values of v vary slightly, but progressively, from the values given by the atomic number relation.

It will be seen that in the equation

$$\frac{2^{1/2}h^{3/2}c^{1/2}}{m^{1/2}\lambda^{1/2}e\pi} = 2e(N - 3.6)$$

a variation in m can affect only the left-hand member of the equation, and that the only other quantity in this member which may possibly vary (since λ is measured) is the quantity h, or, in other words, the possible variable quantity in this member is the fraction h^3/m. Since the right-hand member of the equation merely shows that Q increases by $2e$ in passing from any element to the one of next higher atomic number, there is no reason for thinking that this law changes and that the increase of charge becomes variable for elements of higher atomic number than 58.

If we may rely upon the constancy of the right-hand member of the equation, we may compute values of h^3/m for each of the given wave-lengths and perhaps decide whether the variation in this side of the equation is due wholly to m, or whether h is also probably variable.

In Table I. Blake and Duane's wave-length measurements are given in the third column, those of Siegbahn and Jönssen in the fourth column, the wave-lengths calculated from our formula in the fifth column and the values of h^3/m required to give the measured wave-lengths in the sixth column. It may be seen that h^3/m remains constant to within the errors of measurement to about $N = 58$, from which point on it decreases, while the calculated values of the wave-length also show a systematic variation from about the same value of N.

When these values of h_3/m are plotted as ordinates against their corresponding wave-lengths as abscissas a smooth curve is obtained from which values of h^3/m may be taken for any wave-length down to $\lambda = 0.1 \cdot 10^{-8}$. If corresponding values of v could now be found it would be possible to calculate the respective values of h. This I have attempted to do in the following way:

All of the wave-length calculations and the values of h^3/m given in Table I. are based upon the assumption that the speeds of electrons expelled under the stimulus of ultra-violet light or of X-rays are the orbital speeds which they possessed at the instant of breaking away from their respective atoms. If this be the true explanation of their velocities, it seems almost certain that the same explanation must account for the speeds of the β-particles expelled by radioactive bodies. This would

TABLE I.

Element.	N.	$\lambda 10^8$, B. & D.	$\lambda 10^8$, S. & J.	$\lambda 10^8$, Calc.	h^3/m, 10^{54}.
Br.....	35	.9179		.924	309.5
Rb.....	37	.8143		.817	310.5
Sr.....	38	.7696		.771	311
Yt.....	39	.7255		.727	309.5
Zr.....	40	.6872		.689	311
Nb.....	41	.6543		.654	311
Mb.....	42	.6180		.618	311
Ru.....	44	.5584		.558	311
Rh.....	45	.5324		.532	311.5
Pd.....	46	.5075		.507	312.5
Ag.....	47	.4850		.484	312
Cd.....	48	.4632	.4629	.463	312
In.....	49	.4434		.443	312.5
Sn.....	50	.4242	.4231	.424	312
Sb.....	51	.4065		.409	312
Te.....	52	.3896	.3877	.389	310.5
I.....	53	.3737	.3715	.373	310.5
Cs.....	55	.3444	.3436	.345	310.5
Ba.....	56	.3307	.3306	.332	310.5
La.....	57	.3188	.3186	.320	310.5
Ce.....	58	.3073	.3064	.308	310.5
Pr.....	59		.2946	.297	309
Nd.....	60		.2835	.286	308
Sa.....	62		.2636	.2665	307.5
Eu.....	63		.2543	.2585	306.5
Gd.....	64		.2456	.2495	306.5
Dy.....	66		.2294	.234+	305.5
Ho.....	67		.2214	.2265	304.5
Pt.....	78		.1578	.165	298.5
Au.....	79		.1524	.1605	296.5
Hg.....	80		.1479	.156	295
Tl.....	81		.1427	.152	292
Pb.....	82		.1385	.148	291.5
Bi.....	83		.1346	.1445	290
Th.....	90		.1127	.122	288
U.....	92		.1048	.1167	280

require that the Einstein photoelectric equation should hold for the energies of the β-rays and the frequencies of the accompanying γ-radiations from the same elements. Rutherford and his colleagues have given much time to the investigation of this question, and are of the opinion that such a relation does, indeed, hold and that some of the γ-rays from Ra-B and Ra-C are the true K- and L-radiation bands of these elements.

In an article on The Connexion Between β and γ Ray Spectra[1] Ruther-

[1] Phil. Mag., 28, 305–319.

ford gives the velocities relative to the velocity of light of fourteen groups of β-rays expelled from Ra-B and ten groups of β-rays from Ra-C. In an article by Rutherford and Andrade on The Spectrum of the Penetrating γ Rays from Radium B and Radium C in the same volume of the Philosophical Magazine is given a table of the wave-lengths of fourteen different lines in the γ-ray spectrum of these elements. The authors do not know to which element the various lines belong, but give their reasons for believing that the shortest wave-lengths are due to Radium C, and that the lines $\lambda = 1.59 \cdot 10^{-9}$ and $\lambda = 1.69 \cdot 10^{-9}$ are the characteristic K-radiation lines of Radium B.

In the data on the velocities of β-rays the groups of rays of speeds 1.905–2.100 are marked very strong, indicating that there are more β-rays of this speed than of any other unless it be 1.01. Since the lines of wave-length 1.59 and 1.69 are referred to as the strongest lines in the γ-ray spectrum, it seems reasonable to assign these lines to the group of β-rays whose speeds are from 1.9 to $2.10 \cdot 10^{-10}$.

If we calculate a value of m for a speed of $1.9 \cdot 10^{-10}$ we find $m = 1.16 \cdot 10^{-27}$. Since the wave-length 1.69 lies within the spectral region for which we have computed the value of h^3/m, we may take the value of this ratio from the curve, which was drawn for that region, where we find it to be $300 \cdot 10^{-54}$. Accordingly, $h^3 = 348 \cdot 10^{-81}$ and $h = 7.03 \cdot 10^{-27}$.

In order to express graphically the relation of h to m we must have this relation at one more point, at least, since we already have the initial values of the two quantities. By a slight extrapolation of our curve it was seen that the γ-ray line $\lambda = .99$ must give $h^3/m = 276$, approximately. For trial purposes this line was assigned to the highest speed β-particle from Radium-B, viz., $v = 2.469$. For this speed $m = 1.2 \cdot 10^{-27}$, and if $h^3/m = 276$, $h = 7.60$.

When these three values of h and m were compared they seemed to suggest that h varies as $\log m$. On plotting a curve with the values of h as abscissas and the values of $\log m$ as ordinates, a linear relation was seen to hold between the two quantities, so that for $h \cdot 10^{27}$ and $\log m \cdot 10^{28}$, $h = 4.25 \log m + 2.50 \cdot 10^{-27}$.

Assuming the validity of this equation it is possible to calculate m and h for each of the speeds given, and to compare the values of h^3/m obtained in this way with values taken from our curve for each of the wave-lengths measured in the spectra of Radium B and Radium C. This is done in Table II., where the velocities of the β-rays from Radium B are given in the first column, the corresponding values of m as computed from the equation $m = m_0/\sqrt{1 - (v^2/c^2)}$ in the second column, h as derived from

the equation $h = 4.25 \log m + 2.50$ in the third column and h^3/m in the fourth column. In the fifth column are given values of λ corresponding to the values of h^3/m in the fourth column as taken from the curve drawn to show this relation. In the sixth column are given the values of λ as measured by Rutherford and Andrade in the γ-radiation of Radium B and Radium C.

TABLE II.

β Radiation from Radium B.				γ Radiation.	
v.	m.	h.	h^3/m.	λ Calc.	λ Meas.
					.71
2.469	1.59	7.60	276	.99	.99
2.450	1.56	7.55	278	1.03	——
2.391	1.49	7.46	281	1.10	——
2.361	1.46	7.45	282	1.15	1.15
2.286	1.39	7.35	287	1.24	——
2.252	1.36	7.31	288	1.27	——
2.193	1.32	7.25	289.5	1.31	——
2.157	1.295	7.22	292	1.37	1.37
2.100	1.26	7.16	292	1.37	1.37
1.968	1.195	7.06	297	1.55	1.59
1.905	1.165	7.03	300	1.69	1.69
——	——	——	——	——	1.96
——	——	——	——	——	2.29
——	——	——	——	——	2.42
1.278	.995	6.74	307	2.62	2.62
1.242	.990	6.73	308	2.90	2.96
——	——	——	——	——	3.24
——	——	——	——	——	3.93
1.011	.956	6.66	309	4.28	4.28

A comparison of columns five and six of Table II. will show that nine of the fourteen measured wave-lengths correspond as closely as could be expected with the values calculated from the speeds of the β-particles on the assumption that $h = 4.25 \log m + 2.50$. For the three slowest groups of β-particles the values of h^3/m fall on a part of the curve where h^3/m varies but slightly with the wave-length, and where the values of λ derived from this curve must be more uncertain than for the faster β-particles.

The slowest speed given has been assigned to the wave-length 4.28 from the following considerations: The radiation of particles at this speed is marked "very strong," indicating that this group of β-particles corresponds to a characteristic radiation of the element. I have shown elsewhere[1] that the shortest wave-length of the characteristic L-radiation

[1] Electrical Charges of Atoms and Ions, Leland Stanford Junior University Publications, p. 70.

may be calculated from the equation

$$\lambda = \left[\frac{2.882 \cdot 10^{-12}}{e(N - 15.4)}\right]^2$$

for elements up to atomic number 71. This equation would give for Radium B ($N = 82$) a wave-length $\lambda = 4.33$, which is within the error of measurement the line 4.28.

The measured speeds of the β-particles from Radium C are with two exceptions much higher than those from Radium B, and the curve derived from X-ray wave-lengths may not safely be extrapolated far enough to give a probable estimate of their γ-ray wave-length. The slowest group of these particles has a velocity of $2.25 \cdot 10^{10}$, which would make its γ-ray wave-length about 1.27 from our computation. The next slowest group of β-particles has a speed of 2.604. This would give $m = 1.80$ and $h = 7.83$, making $h^3/m = 266$. This seems to correspond with the measured wave-length .71.

It seems not improbable that the γ-radiation from the other β-particles of Radium C was of too short wave-legth to be measured by Rutherford and Andrade. With their spectrometer the wave-length taken as .71 was determined from a deviation of only 43′, and the authors say, "It is surprising that the architecture of the crystal (rock salt) is sufficiently definite to resolve such short wave-lengths. This is especially the case when we consider that owing to the heat agitation of the atoms, the distance between the atoms must be continually varying over a range comparable with the wave-length of the radiation."

In a paper by F. Dessauer and E. Back published in Berichte der Deutschen Physikalischen Gesellscahft, May 30, 1919, the authors give the results of their attempt to find the shortest wave-length given by X-rays from any element. By using a potential of 170 k.v. and an exposure of 9 hours they found the apparent limit of X-rays at $\lambda = .57 \cdot 10^{-9}$. It is probable that all of the β-rays from Radium C except the two sets referred to would be accompanied by γ-rays of shorter wave-length than $.57 \cdot 10^{-9}$.

If these estimates of the relation between the speeds of β-rays and the wave-lengths of their accompanying γ-radiations are correct, it seems almost certain that the speeds of β-particles are the orbital speeds which they possessed at the instant of escaping from the atoms.

It also seems certain that for speeds as great as those of β-particles the product $\frac{1}{2}mv^2$ of the β-rays does not bear a constant ratio to the frequency of the accompanying γ-radiation.

It has already been mentioned that Duane and his colleagues have

calculated values of v from the K-radiation of a large number of elements which values are related to the atomic numbers by an equation of the form of Moseley's equation for the square root of the frequencies. It will be seen from our fundamental equation for the atomic charge that this will necessarily be true if h^3/m is constant or bears a straight line relation to the atomic numbers. It was observed, however, that this law of Duane's holds only so far as the wave-length may be correctly calculated on the assumption of the constancy of h^3/m.

In order to test the same relation with values of v calculated from our hypothesis, I have calculated values of m for a series of values of v from $v = 0.1 \cdot 10^{10}$ to $v = 2.2 \cdot 10^{10}$, letting

$$m = \frac{9 \cdot 10^{-28}}{\sqrt{1 - v^2/c^2}}.$$

Then from the equation $\frac{1}{2}mv^2 = \nu(4.25 \log m + 2.50)$ I have calculated values of ν for each value of v used in computing m. These values of ν were then plotted on a large scale as abscissas against the respective values of v as ordinates. From this curve it is possible to obtain a value of v accurate to the third significant figure for any value of ν from $\nu = 0.1 \cdot 10^{18}$ to $\nu = 40 \cdot 10^{18}$.

Values of v corresponding to all the K-radiation frequencies measured by Siegbahn and Stenström, Siegbahn and Jönssen, Blake and Duane, Duane and Hang-Fuh-Huh and Duane and Shimizu were then taken from this curve and plotted against their respective atomic numbers. In the case of the Siegbahn and Stenström measurements the frequencies for the $K\beta$ lines were used throughout. It was found that the velocities calculated in this manner bear a straight line relation to the atomic numbers with no tendency to systematic departure on either side of the line for higher atomic numbers. This relation of velocity to atomic number may be expressed by the equation $v = 2.054(N - 1.66)$.

The relation between the values of v calculated in the two ways is shown in Table III., in which the wave-length measurements of Siegbahn and his colleagues are given in the third column and those of Duane and his colleagues in the fourth column. The corresponding values of ν are given in the fifth column. Where two values of λ are available a mean value is used in computing ν. The sixth column gives v from the equation

$$v = \sqrt{\frac{2\nu(4.25 \log m + 2.50)}{m}}$$

and the seventh column gives v from the equation $v = 2.054(N - 1.66)$.

It will be seen that though the series runs to $N = 92$, the agreement

TABLE III.

$$v_1 = \sqrt{\frac{2\nu(4.25 \log m + 2.50}{m}},$$

$$v_2 = 2.054(N - 1.66).$$

Element.	N.	λ Siegbahn and Others.	λ Duane and Others.	ν.	v_1.	v_2.
Mg.....	12	9.477		.3165	.213	.2124
Al.....	13	7.986		.3753	.233	.233
Si.....	14	6.759		.4436	.2535	.2535
P.....	15	5.808		.516	.274	.274
S.....	16	5.018		.5978	.295	.2946
Cl.....	17	4.394		.6826	.316	.3152
K.....	19	3.449		.8700	.357	.3564
Ca.....	20	3.086		.973	.377	.377
Sc.....	21	2.778		1.080	.3975	.3972
Ti.....	22	2.509		1.195	.418	.418
Va.....	23	2.281		1.315	.437	.438
Cr.....	24	2.079		1.443	.458	.458
Mn.....	25	1.902	1.8892	1.582	.480	.4795
Fe.....	26	1.743	1.7396	1.721	.500	.500
Co.....	27		1.6018	1.86	.520	.521
Ni.....	28		1.4890	2.015	.540	.541
Cu.....	29		1.3785	2.175	.560	.562
Zn.....	30		1.2963	2.315	.578	.582
Ga.....	31		1.1902	2.521	.602	.603
Ge.....	32		1.1146	2.693	.623	.623
As.....	33		1.0435	2.875	.644	.644
Se.....	34		.9790	3.065	.665	.664
Br.....	35		.9179	3.268	.685	.685
Rb.....	37		.8143	3.684	.726	.726
Sr.....	38		.7696	3.898	.745	.746
Yt.....	39		.7255	4.136	.767	.767
Zr.....	40		.6872	4.365	.788	.788
Nb.....	41		.6503	4.61	.808	.808
Mo.....	42		.6180	4.855	.830	.829
Ru.....	44		.5584	5.374	.871	.871
Rh.....	45		.5330	5.63	.891	.890
Pd.....	46		.5075	5.913	.912	.911
Ag.....	47		.4850	6.185	.9325	.932
Cd.....	48	.4629	.4632	6.48	.953	.952
In.....	49		.4434	6.766	.973	.973
Sn.....	50	.4231	.4242	7.08	.994	.994
Sb.....	51		.4605	7.384	1.014	1.014
Te.....	52	.3877	.3896	7.722	1.036	1.034
I.....	53	.3715	.3737	8.065	1.057	1.055
Cs.....	55	.3436	.3444	8.725	1.098	1.096
Ba.....	56	.3306	.3307	9.08	1.118	1.116
La.....	57	.3186	.3188	9.42	1.138	1.137
Ce.....	58	.3064	.3068	9.80	1.160	1.157
Pr.....	59	.2946		10.18	1.181	1.178

Element.	N.	λ Siegbahn and Others.	λ Duane and Others.	ν.	ν_1.	ν_2.
Nd.....	60	.2835	.2861	10.54	1.201	1.199
Sa.....	62	.2636		11.38	1.243	1.239
Eu.....	63	.2543		11.79	1.263	1.260
Gd.....	64	.2456		12.21	1.284	1.282
Tb.....	65		.2398	12.51	1.299	1.301
Dy.....	66	.2294	.2308	13.04	1.324	1.322
Ho.....	67	.2214		13.54	1.348	1.343
W.....	74		.1786	16.78	1.485	1.486
Os.....	76		.1683	17.82	1.531	1.528
Pt.....	78	.1578		19.01	1.572	1.568
Au.....	79	.1524	.1541	19.57	1.590	1.589
Hg.....	80	.1479	.1501	20.14	1.608	1.608
Tl.....	81	.1427		21.01	1.640	1.626
Pb.....	82	.1385	.1424	21.35	1.651	1.650
Bi.....	83	.1346		22.29	1.681	1.671
Th.....	90	.1127		26.61	1.814	1.815
U.....	92	.1048		28.60	1.866	1.835

between the two values of v is very close and no progressive variation appears. In only one case, that of uranium, is the difference between the two values of v as great as one per cent. From the equations here used the wave-length of the shortest K-radiation from uranium should be $\lambda = .1099 \cdot 10^{-8}$ instead of $.1048 \cdot 10^{-8}$, as given by Siegbahn and Jönssen. Dessauer and Back, in the paper already mentioned, attempted to find the wave-length of the $K\beta$ line of uranium, and concluded that it lies between $.104 \cdot 10^{-8}$ and $.154 \cdot 10^{-8}$. When extrapolated from their measured wave-length of the platinum line by Moseley's equation they found it to be between .114 and $.136 \cdot 10^{-8}$, but it is well known that Moseley's equation does not apply closely to elements of the higher atomic numbers. From Sommerfeld's formula this line would lie between .131 and $.146 \cdot 10^{-8}$.

The one other considerable deviation is in the case of thallium, for which our equation would give a wave-length $.1455 \cdot 10^{-8}$ instead of $.1427 \cdot 10^{-8}$, as determined by Siegbahn and Jönssen.

On the whole, the two values of v are in very satisfactory agreement when we take into consideration that the most careful wave-length determinations in this region by different experimenters frequently differ in the third decimal place.

While it is probably too much to claim that the relations shown in Table II. and Table III. give an adequate test of the value of the expression here substituted for h in the equation $\frac{1}{2}mv^2 = h\nu$, they are, so far as is known to the writer, the only tests thus far made of the validity of

this equation. The equation $Ve = h\nu$ has frequently been tested by experiment, but the assumption that all the energy given to an electron in an electrostatic field appears in the form of kinetic energy of the electron has not been so tested. Whatever evidence may be derived from the relations here shown indicates that in this form h does not represent an energy quantum, but only a function of the mass of the electron.

A STUDY OF THE RESIDUAL IONIZATION IN A GAS WITH REFERENCE TO TEMPERATURE EFFECTS.

By C. H. Kunsman.

Synopsis.

A Study of the Residual Ionization in a Gas with Reference to Temperature Effects.—A form of electrometer was constructed which used the principle of the Wulf electrometer in separating the conduction of charge across the insulation from the conduction in the gas due to ionization. *The new features of this apparatus* were that it was possible (1) to make a quick and accurate test of the insulation properties under the same conditions as those under which the ionization tests were made, and (2) to subject the instrument to a wide range of temperatures.

Ions generated per C.C. per Second.—The data experimentally determined show that the number of ions generated per c.c. per second within an air tight chamber is 8.22 in the basement of the Physical Laboratory and 4.15 over the Pacific Ocean.

Residual Ionization not due to Thermal Impact.—The results of tests between $-44.6°$ C. and $92.5°$ C. show no indications that the residual ionization is due in any part to a molecular impact of thermal agitation.

Effect of Temperature on Insulation System.—Observations made of changes in temperature on the insulation system, would seem to account for the apparent daily and seasonal variations of the residual ionization as previously reported.

Conclusions from Observations at Low Temperatures.—At temperatures from $-30.5°$ C. to $-44.6°$ C. an increase in conduction across the insulation system was noticed, which was comparable to the apparent increase in ionization as reported at high altitudes. It would therefore seem that it is not necessary to assume, as has been done by previous observers, that there is a highly radioactive cosmic layer in the upper atmosphere; or that the sun is a source of penetrating radiation sufficient to generate 90 ions per c.c. per second.

I. Introduction.

THE experiments described in this paper were undertaken with the purpose of studying the ionization of a gas as observed in an air-tight vessel. This was done with a twofold purpose: (1) to find an accurate method of measuring this conductivity in the gas, and (2) to make a thorough investigation of the effect of temperature on this conductivity.

C. T. R. Wilson[1] and Geitel[2] were the first to notice that gases in closed vessels conducted electricity and were therefore ionized. The more recent experiments of Simpson and Wright,[3] King,[4] Murray,[5]

[1] Wilson, Proc. Camb. Phil. Soc., II., 52, 1900.

[2] Geitel, Phys. Zeit., 2, 116, 1900.

[3] Simpson and Wright, Roy. Soc. Lon. Proc., 85, 175, 1911.

[4] King, Phil. Mag., 26, 610, 1913.

[5] McLennan and Murray, Phil. Mag., 30, 430, 1915.

McLeod,[1] McLennan,[2] Wright,[3] and the author[4] showed conclusively that ions are being generated in an airtight metal chamber, as free from radio-active substances as possible, at the rate of from 8 to 9 ions on land, and over the ocean and large lakes from 4 to 5 ions per c.c. per second. These results include observations made over practically every continent and ocean. Some observations indicate a pronounced variation. Lassalle[5] notices a diurnal variation in the Philippine Islands, while Gockel[6] finds the value for the winter months of December and January lower by 1 to 1½ ions per c.c. per second than for the months of June and July.

In connection with the ionization within a closed chamber, we are confronted with two problems: (1) to find the source of the residual ionization of from 4 to 5 ions which is obtainable when the chamber is screened naturally over large bodies of water, or artificially on land; (2) to find an explanation for the very large increase in the ionization at high altitudes, as observed in a balloon by Hess[7] and Kolhörster.[9] The latter reports an increase of from 4.3 ions at 2,000 meters to 80.4 ions at 9,000 meters, over the values obtained at the earth's surface. Treleaven[9] concludes that the residual ionization of 4 ions is due to α, β, and possibly γ radiation emitted from the walls of the chamber. Murray[5] comes to the same conclusion from observations of this effect when the walls of the chamber were of ice. Kingdon[10] shows that a part of the residual ionization is due to thermal agitation and derives a formula which agrees with his experimental results.

The only other experimental works are those of Patterson[11] and Devik.[11] Patterson used a large iron chamber containing an electrode insulated by ebonite, and measured the current with an electrometer. He got a value of 61 ions, so it would be doubtful if he could observe a small temperature effect at ordinary temperatures. Devik attempted to measure the current at the instant of greatest compression in an adiabatic compression and found no effect at ordinary temperatures.

[1] McLennan and McLeod, Phil. Mag., Oct., 740, 1913.
[2] McLennan, PHYS. REV., 26, 526, 1908.
[3] Wright, Phil. Mag., 17, 295, 1909.
[4] Kunsman, PHYS. REV., 6, 493, 1915.
[5] Lassalle, PHYS. REV., 5, 135, 1915.
[6] Gockel, Phys. Zeit., 16, 350, 1915.
[7] Hess, Phys. Zeit., 14, 610, 1913.
[8] (*a*) Kolkörster, Phys. Zeit., 14, 1153, 1914; (*b*) Ber. Deut. Phys. Ges., 13, 721, 1914.
[9] Treleaven, Phil. Mag., 30, 427, 1915.
[10] Kingdon, Phil. Mag., 32, 397, 1916.
[11] Patterson, Phil. Mag., 6, 231, 1903.
[12] Devik, Sitz. d. Heid. Akad. Wiss., 24, 1914.

The question has been theoretically investigated by Langevin and Rey,[1] and by Wolfke.[2]

Results obtained by the author seem to throw light on each of the following questions: (1) Do molecules of gases under ordinary temperatures collide in such a way as to produce ionization? (2) Is it necessary to assume a source of ionization in the upper atmosphere sufficient to produce ions at the rate of about 90 per c.c. per second?

II. APPARATUS.

Preliminary experiments on the measurements of the natural ionization at the State College of Pennsylvania, at the University of California,[9] and on the Pacific Ocean between San Francisco and Los Angeles, were made with a Spindler and Hoyer[10] aluminum leaf electroscope designed for radioactive tests when the gases tested could be drawn into the ionization chamber. Two chambers of different sizes were essential for the method used.

More extensive tests were made by means of an electrometer designed by the author which could be subjected to a wide range of temperature. The object of this was to make it possible to measure ionization down to the temperature of liquid air. The main feature of the instrument was that it provided an easy test for leak across the insulation system without altering the conditions under which the tests were made. Such a test of conduction over the insulation could not be made on the apparatus used by the observers previously mentioned.

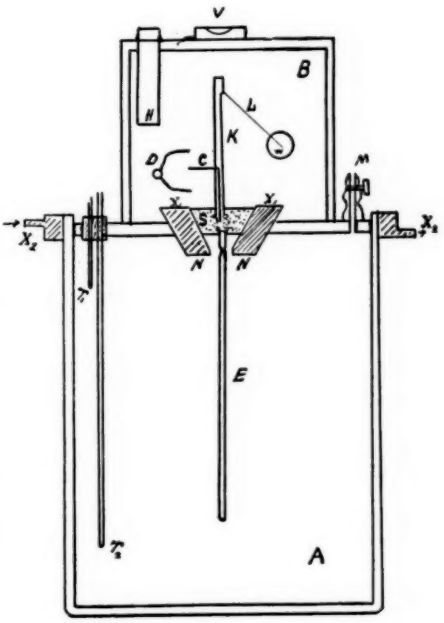

Fig. 1.

Practically the entire apparatus was built of as pure zinc as obtainable, since this metal seems more free from radioactive impurities than others. The ionization chamber *A* consisted of a zinc cylinder about 20 cm. long and 10 cm. in diameter, Fig. 1, the walls of the cylinder

[1] Langevin and Rey, Le Radium, 10, 142, 1913.

[2] Wolfke, Le Radium, 10, 265, 1913.

being $2\frac{1}{2}$ mm. thick. Into this screwed the upper part which consisted of a zinc box B, forming the walls of the electrometer. The sulphur plug S holds in place the zinc electrode E, and another zinc rod K, upon which the aluminum leaf L was attached. A very small rod C moves up or down through the center of the zinc rod K. The rod C is raised or lowered by an insulated rod D which passes through a ground glass joint attached to the electroscope case. The volume of A or B can thus be electrically connected or separated as desired. The electric charge is communicated to the insulated system by means of the same rod D. After the system is charged, rod D is disengaged from rod C and grounded, as is also the case of the electrometer. An external dial on the case and an indicator on the charging rod assure the rod D being turned to the same position after each time the instrument is charged.

The sulphur insulation was kept cool or warm as was necessary by means of a system X_1 through which water or alcohol was run. A similar cooling system X_2 kept the wax joints cool.

Chamber B was kept as dry as possible by means of metallic sodium, contained in a receiver H. Thermo-couples T_1 and T_2 made it possible to measure the temperature of the gas quickly and accurately. A stopcock M permitted the ionization chamber to be exhausted and filled with dry gas. A spirit level V made it possible to keep the instrument always in the same position. A very fine quartz fiber was attached to the end of the leaf and the position of the fiber observed through windows by means of a telescope. The quartz fiber greatly increased the accuracy of the observations.

The zinc electrode E could be easily removed and a small cup containing sodium could be sealed to the bottom of the water jacket N, so that the loss of charge due to the volume B together with the total leakage across the insulation support could be determined. The volume of chamber A was 1,277 c.c. The electrostatic capacity of the electrometer as determined with two spheres of different radii,[1] was 4.48 cm. with the cup at N in place, and 6.50 cm. with the cup removed and the central electrode replaced. The drop in potential for a given number of scale divisions was obtained from the calibration curve, Fig. 2.

III. THEORY.

If C is the electrostatic capacity of a body, which is charged to a potential E, and after a certain time some of the charge has disappeared, it is evident that we can express this loss, or change of quantity, of electricity as $\Delta Q = C\Delta E$; where ΔE is the drop in potential for a unit of

[1] Lichtenecker, Phys. Zeit., 13, 516, 1913.

time Δt. This loss of charge may be thought of as a neutralization of N ions, each carrying the unit charge, e. If n is the number of ions formed per c.c. per second in the volume V in the time Δt, we may write $N = nV\Delta t$. It follows that $\Delta Q = C\Delta E = NE = n\Delta teV$, or $n = C\Delta E/Ve\Delta t$.

Now ΔE is made up of two parts (A) a conduction of charge across the insulation support and (B) a loss of charge due to ions being formed in the gas, which neutralizes some of the charge on the insulated system. If two different volumes of gas are taken in such a way as not to alter the conduction of charge over the support, two equations can be set up of the form, $Q_1 = A + BV_1$ and $Q_2 = A + BV_2$; where Q_1 and Q_2 are the loss of charge per second with volumes V_1 and V_2 respectively and where $B = ne$. From this we get

$$n = \frac{C_1\Delta E - C_2\Delta E}{(V_1 - V_2)e},$$

which is the formula used for all calculations of n.

IV. First Investigation.

In the first set of experiments dry, dust-free air was introduced in the ionization chamber of the Spindler and Hoyer electroscope, in which the amber insulation was replaced by sulphur. Observations were taken of the rate of fall of the leaf, or loss of charge, after the rate of loss became constant. An apparent soaking in of the electric charge was shown by the relatively rapid fall just after the instrument was charged. This effect lasted about two hours. Quite a pronounced increase in ionization was always noticed immediately after the instrument was filled with air. This effect largely disappeared in about 4 days, which leads one to conclude that it was due to radium emanation, its half period being 3.8 days. The rate of fall was observed for about 4 weeks for each ionization chamber, and a mean value of n calculated. The mean value of n as obtained in the basement of the physical laboratory was 8.68 ions. Variations in this number were observed. In the light of recent experiments, these variations can be attributed to a change in the temperature of the instrument and were not due to a change in ionization.

In a test made the same year (1915) with the same apparatus, the value of n on the Pacific Ocean, between San Francisco, and Los Angeles was found to be 4.15 ions. In the calculation of the results of the observations over the ocean, the leak over the support was calculated from data obtained on land. The differences between the observed loss of charge on the ocean, and the calculated leakage across the support, as observed on land, was attributed to the formation of ions.

V. SECOND INVESTIGATION.

The zinc electrometer was used in all experiments where the temperature of the gas was varied. The instrument was made airtight by covering all possible sources of leak with sealing wax and finally applying a number of coats of shellac. The ionization chamber was filled with air or carbon dioxide which was made as free from moisture as possible by drawing the gas through calcium chloride and phosphorus pentoxide tubes, and finally bubbled through two bottles of concentrated sulphuric acid. The thermocouples, previously annealed and calibrated from the temperature of liquid air to the melting point of lead, were sealed into a hard rubber plug with sealing wax. This plug was then screwed into the top wall of the ionization chamber, and covered with more sealing wax. The other junctions of the thermo-couples were kept in a bath of melting ice. The E.M.F. was measured on a potentiometer in microvolts. In practically all tests the instrument was charged in the even-

TABLE I.

Length of Test.	Chamber *A* Connected, Disconnected or Cup in Place.	Average Temp. of Gas °C.	Rate Scale Div. per Min.	Rate Volts per Min.	n (Calculated).	Remarks.
			Air.			
14 dys.	Conn.	21 to 24°	.0915	.02955	8.22	Instrument surrounded with felt.
10 hrs.	Conn.	" "	.0931	.03007	8.55 (max.)	Instrument surrounded with felt.
8 hrs.	Conn.	" "	.0896	.02914	7.88 (min.)	Instrument surrounded with felt.
5 dys.	Cup in place	" "	.0703	.02271		Instrument surrounded with felt.
10 dys.	Disconn.	" "	.0336	.01085		Instrument surrounded with felt.
20 hrs.	Conn.	−12.5	.0721	.02603	6.20	Chamber immersed in ice and salt.
10 hrs.	Disconn.	−12.5	.0298	.01076		Chamber immersed in ice and salt.
48 hrs.	Conn.	22.0	.0815	.02632	6.37	Chamber immersed in brine.
5 hrs.	Disconn.	22.0	.0327	.01056		Chamber immersed in brine.
5 hrs.	Conn.	34.0	.0905	.02920	8.04	Chamber in heater.
			Carbon Dioxide.			
7 dys.	Conn.	21 to 24°	.1104	.03566	11.95	Chamber in heater.
6 hrs.	Conn.	38.8°	.0985	.03171	9.58	Chamber in heater.

ing beyond the field of view of the telescope so that by morning it was within view. This did away with irregularities previously mentioned.

The residual ionization was determined at room temperature for air and carbon dioxide. The results for carbon dioxide are somewhat higher than those obtained by Treleaven. Since commercial carbon dioxide was used the increase may be attributed to radioactive impurities in the gas. By means of the formula for n, the given constants of the apparatus and the data given in Table I., the resulting values of n were calculated.

Tests were undertaken at low temperatures for two reasons: (1) if ionization is produced by molecular impact due to thermal agitation, such an effect could best be investigated by decreasing this molecular motion as much as possible, (2) there is no record of such tests being made at low temperatures.

The electrometer was clamped rigidly in a frame and a Dewar flask containing solid carbon dioxide and alcohol raised until the ionization chamber was completely surrounded with the cooling solution. During this change of temperature of the gas within the chamber, very pronounced variations of the rate of leak were observed. When the warm instrument first came in contact with the cold solution, the divergence of the leaf increased a few scale divisions. However, in a few minutes the leaf came back to its original position, and quite a rapid rate of fall of the leaf was observed. For some time, depending upon how rapidly the temperature was lowered, the rate of leak appeared to be normal. With a further decrease in temperature the rate of loss of charge became quite rapid, and with a further lowering of temperature the charge on the insulated system had disappeared entirely. Table II. shows how this leak depends upon the temperature.

The central electrode was then disconnected from the charged system and a similar set of results obtained, also given in Table II. This at once shows us that the increased conductivity takes place across the insulation. Repeated tests with additional precautions against air leaks were taken, and in every case the same effects were noticed. Separate insulation tests were made on sulphur, amber and hard rubber, in order to determine whether the increased conductivity at low temperatures was due to a property of the insulator or was due to the presence of water vapor. The insulators were attached to wires and were sealed into glass tubes. In some cases the tubes were exhausted and filled with air which had been passed through calcium chloride, phosphorous pentoxide, and concentrated sulphuric acid. No noticeable increase in the conduction was observed at the temperatures of $-72°$ and $-187°$ C., when the air was carefully dried. But in cases where water vapor

TABLE II.

Length of Observation.	Average Temp. of Gas ° C.	Rate Scale Div. per Min.	n.*	Remarks. Carbon Dioxide in Chamber-connected.
30 min.	22.0	.1010	10.0	In alcohol bath at room temperature.
18 min.	−30.5	.1385	20.0	No warming bath used.
50 min.	−34.5	.1310	18.6	No warming bath used.
30 min.	−29.0	.1265	17.6	No warming bath used.
40 min.	−36.5	.1250	17.4	No warming bath used.
30 min.	−41.0	.1785	28.8	No warming bath used.
10 min.	−41.2	.2778	50.3	No warming bath used.
6 min.	−43.2	1.0820	222.5	No warming bath used.
1½ min.	−44.6	5.4800	1,162.0	No warming bath used.
½ min.	−44.6	7.0500	1,498.0	No warming bath used.
4 min.				Small stream of alcohol used for warming bath.
7 min.	−38.9	1.0000	204.0	No warming bath used.
2 min.	−42.3	4.1000	867.0	No warming bath used.
6 min.				Small stream of alcohol used for warming bath.
7 min.	−38.8	.8560	181.5	No warming bath used.
10 min.	−39.2	.8550	181.0	No warming bath used.
20 min.	−40.1	.6250	124.5	No warming bath used.
25 min.	−36.2	.2320	40.5	No warming bath used.
				Air in Chamber-connected.
1 hr.	24.5	.0938	8.68	No warming bath used.
45 min. later	−41.7	charge lost		No warming bath used.
30 min.	+20.0	.1140		Instrument recharged (25 min. later).
8 min.	−31.1	.1250	17.4	Warming bath used.
14 min.	−31.1	.1570	25.1	Warming bath used.
11 min.	−35.2	.1820	29.7	Warming bath used.
10 min.	−33.4	.2000	33.4	Warming bath used.
11 min.	−25.8	.1630	25.5	Warming bath used.
37 min.	−28.1	.162		No warming bath.
15 min. later	−40.2	charge lost		No warming bath.
				Air in Chamber-disconnected.
2 hrs.	22.5	.0337		Room temperature.
45 min.	−21.4	.0248		Warming bath used.
36 min.	−30.1	.0694		No warming bath used.
20 min.	−29.6	.0588		No warming bath used.
8 min.	−34.5	.1250		No warming bath used.
5 min.	−36.4	.6450		No warming bath used.
1 min.	−41.0	3.2500		No warming bath used.
	−41.0	charge lost		No warming bath used.

* n calculated upon the supposition that the increased rate of loss of charge is due to ionization within the gas.

was present an increased conduction took place. The rate of increase depended upon the insulator and form of the electrode. Insulation tests were also made above room temperature. Sulphur proved to be

the best insulator under all conditions. Amber seemed to be especially sensitive to traces of moisture.

It was found that more even temperature conditions could be obtained at these low temperatures by allowing a small stream of alcohol at room temperature to flow through the system designed to keep the insulation cool when the gas was heated to higher temperatures.

As seen from Table II., there is a very marked increase in conduction across the insulation support when the average temperature of the gas is from − 15° C. to − 30° C. and at a temperature of about − 41° C. the charge is lost entirely. The temperature of the sulphur, although below zero, as shown by the amount of ice crystals which form on the external surface of the instrument, is not as low as the temperature of the gas, since the insulation part of the instrument is not immersed in the cooling solution.

Kolhörter[18] does not give the definite temperature of the electrometer for various altitudes, but states that when the temperature of the instrument had fallen to − 10° C. (5 to 6 km. above sea level) and below − 10° C. (6 to 9 km.) one would disturb the readings, if the electrometer was touched with the bare hands. In Table III., we have listed Kolhörster's results and probable atmospheric temperature for such elevations.

TABLE III.

Elevation Km.[18b] Above Sea Level.	* Value in 1913.	* Valve in 1914.	† Temp. °C.[21]
0	——	——	24.9
1	− 1.5	——	21.6
2	+ 1.2	——	16.6
3	+ 4.0	+ 4.3	10.3
4	+ 8.3	+ 9.3	3.9
5	+16.5	+17.2	− 2.5
6	+28.7	+28.7	− 8.5
7	——	+44.2	−14.3
8	——	+61.3	−20.9
9	——	+80.4	−27.2
10	——	——	−33.9

* Increase in ionization over value obtained on the earth's surface.

† Mean value of atmospheric temperature based upon sounding balloon observations at Fort Omaha, Nebraska; Indianapolis, Indiana; Huron, South Dakota; and Avalon, California, for the summer season.

[21] Gregg, Month. Weather Rev., 46, 17, 1918.

On comparing Tables II. and III. we see very marked evidence that we are dealing with similar results due most probably to the same cause, and that there are no abnormal ionization effects at great elevations; nor

any additional source of penetrating radiation in that region. There is no need of introducing the theory of a very radioactive cosmic atmosphere at that elevation to explain these results. It also seems very improbable that the sun could be the source of a penetrating radiation sufficient to form 90 ions per c.c. per second within a closed vessel. And if the value of 90 ions represents the number generated at 9 km., might we not expect a much greater increase in ionization at 17 km. elevation where the temperature[1] is about $-84°$ C.? It does not seem at all unreasonable for us to believe that results would be obtained which would be in agreement with those in Table II.

In order to get such large results by experiments with a radioactive substance, it was necessary to place a considerable quantity of a very active material quite close to the chamber. This would lead us to believe that it is very improbable that there would be such quantities of radioactive material in the upper atmosphere as it would require to produce ionization at this rate. Then again experiments on the earth's surface,[2] show that the radioactive content of the atmosphere only contributes from 0.1 to 0.2 ions per c.c. per second, to the residual ionization. It would thus be necessary for the upper atmosphere to be from 450 to 900 times as radioactive as the atmosphere close to the earth's surface.

There seems to be no record of insulation tests being made at these altitudes. The author has found in measurements made with a similar Wulf electrometer obtained from Germany, that there is a very probable air leak when the change from one volume to the other is made, and in order to keep the gas as dry as possible, it is necessary to replace the metallic sodium from time to time. It would seem that the moisture in the closed gas has no noticeable effect on the insulation at ordinary temperatures, but when sufficiently low temperatures are reached minute ice crystals form on the insulation and conduct the charge to the walls of the instrument; again when the enclosed air is sufficiently warm, these crystals vaporize and normal conditions are established. One notices (see Table II.) that at one instance the rate had increased to 7 divisions per minute, and on allowing a small stream of alcohol at room temperature to flow through the water jacket for 4 minutes, the rate was decreased to one division per minute. A similar effect was noticed a little later in the test when a small stream of alcohol running 6 minutes caused the rate to change from 4.1 to 0.856 divisions per minute. No matter how rapidly the loss of charge takes place at low temperatures on bringing the temperature of the instrument back to room temperature the rate of loss of charge again becomes normal.

[1] Nature, 98, 21, 1916.

[2] Chauveay, Le Radium, 10, 18 and 70. 1913.

Other observations were made with the instrument surrounded by a mixture of ice and salt (Table I.) in this case the temperature of the gas was subject to only small variations during the test. Since a pronounced decrease in the loss of charge was noticed, tests were made when the chamber was surrounded with the same brine at room temperature. Differences were observed both when the chamber was connected and disconnected. A search for the difference led the author to calibrate the instrument at the corresponding temperatures. The result was two distinctly different calibration curves as given in Fig. 2. At room-temperature one scale division is equal to 0.232 volts; while at a temperature below zero, one scale division is equal to 0.361 volts. So that on reducing the respective scale divisions to volts per minute (Table I.) the rate of loss of charge is the same, when the gas is at $-12.5°$ C. in the ice and salt solution, or when the gas is at room temperature and the ionization chamber is immersed in salt water. The difference between 8.22 ions at room-temperature (Table I.) and 6.37 ions with the chamber immersed in salt water is due to the screening effect of the salt water.

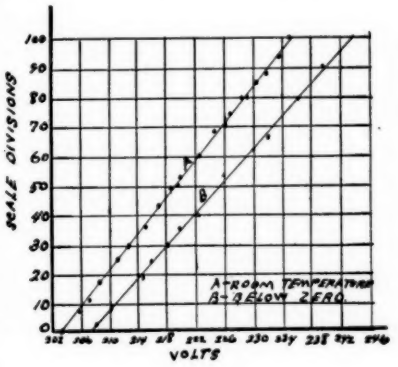

Fig. 2.

For investigations above room-temperature, the ionization chamber was placed in an electric heater formed by winding climax wire about a thin cylindrical sheet of asbestos, and packing this heating coil in magnesia. The heater was made just large enough to contain the ionization chamber. Both the sulphur insulation and the wax joints were kept cool by water jackets. From three fourths to two and one half amperes of current were passed through the heating coil. The results, as given in Table IV., show no increase with temperature below 92.5° C., but from 95 to 100° C. a pronounced increase is noticed, which is again evident when the volume is disconnected, at from 101° C. to 110° C. This shows an increased conduction across the insulation support, and is sufficient to account for the apparent increase in ionization in the gas noted by Kingdon from 80° to 100° C. His only method of testing the insulation was to exhaust the chamber. Thus different conditions were introduced whenever the insulation tests were made.

One notices from Table IV. that from 22° to 92.5° C. and from 22° to 101° C., with volumes connected and disconnected respectively, there

TABLE IV.

Air above Room Temperature.

Chamber Connected.		Chamber Disconnected.	
Temp. ° C.	Scale Div. per Min.	Chamber Temp. ° C.	Scale Div. per Min.
22.5	.091	22.0	.0333
38.0	.089	54.0	.0310
43.0	.088	76.0	.0308
52.0	.090	90.5	.0306
56.0	.087	95.0	.0300
68.0	.087	101.0	.0412
82.0	.079	109.5	.0450
90.0	.085		
92.5	.091		
95.0	.132		
100.0	.130		

is a decided decrease in the rate of loss in scale divisions per minute, instead of an increase as would be the case if there were an increase in the ionization. This change is no doubt largely due to a change in the dielectric constant of the sulphur.

The effect of temperature upon the insulated system can be easily shown by allowing solutions of different temperatures to flow through the water-jacket surrounding the sulphur insulation. When water at room temperature is allowed to flow through there is no change in the position of the leaf. When a solution above room temperature passes through, the leaf at once took a higher position on the scale. When a cold solution was allowed to flow through the water-jacket the leaf immediately took a lower place on the scale than that which it occupied at room temperature. When the temperature of the instrument was again at room temperature the leaf returned to its original position, except for the natural loss of charge which took place in the mean time. The greater part of this change is due to the change in the dielectric constant of the sulphur for various temperatures, as is shown by Schmidt.[1] A small variation may also be due to the change in the flexibility of the aluminum leaf with the change in temperature.

Bearing in mind that it requires a change of about 0.005 scale divisions per minute for an increase or decrease in the generation of one ion per c.c. per second, we find, on referring to Table I. and IV. that we cannot attribute the residual ionization to a thermal agitation of the molecules. Since the apparent variations in ionization can be attributed to temperature effects on the insulation system, it does not seem reasonable to

[1] Schmidt, Ann. der Physik; 44, 335, 1914.

attribute any part of the residual ionization to a temperature effect upon the gas.

VI. Summary.

1. The mean value of the electrical conductivity of air in the basement of the physical laboratory at the University of California was 8.22 ions per c.c. per second when measured in a zinc chamber and 8.68 ions per c.c. per second when measured in a chamber where part of the walls were brass and the other part aluminum.

2. The mean value of the electrical conductivity as measured on the Pacific Ocean was 4.15 ions per c.c. per second.

3. This electrical conductivity, or residual ionization, is not due to molecular impact of thermal agitation.

4. The apparent increase in ionization as observed at high altitudes is solely a temperature effect, and is due to an increase in conduction over the insulation.

5. The effect of changes in temperature on the insulation system of an apparatus of this kind is sufficient to account for the apparent daily and seasonal variations of the residual ionization as reported by some observers, and is not due to a variation of the ionization of the gas within the chamber.

In conclusion I wish to express my thanks for the valuable guidance of Professor E. P. Lewis, under whose direction this work was done; and to Professor E. E. Hall for the laboratory facilities placed at my disposal, and also for helpful suggestions. Finally, acknowledgement is due Mr. W. R. Stamper for help in construction of apparatus.

Physical Laboratory,
University of California,
March 31, 1920.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE ONE HUNDRED AND FOURTH MEETING.

A MEETING of the American Physical Society was held in Denny Hall. University of Washington, Seattle, on June 18, at 9:30 o'clock in connection with a meeting of the Pacific Division, American Association for the Advancement of Science. About thirty members and visitors were in attendance,

The following papers were presented:

A Cable Loaded at One Point—A Special Case of the Catenary. S. H. ANDERSON. (By title.)

The Gibbs Thermodynamical Model. W. P. BOYNTON.

Transparency to Heat Radiation. S. L. BROWN.

Characteristics of Vaccum Tubes. S. L. BROWN AND C. F. NORMAND.

A Study of the Residual Ionization in a Gas with Reference to Temperature effects. C. H. KUNSMAN.

The Continuous Spectrum of Hydrogen in the Schumann Region. E. P. LEWIS.

Multiple Reflection from the Interior of a Ring. A. A. KNOWLTON AND G. A. WATT.

An Optical Illusion. A. A. KNOWLTON AND G. A. WATT.

Velocity of Sound from a Moving Source. R. B. ABBOTT AND J. W. COOK.

Voltage Wave Analysis with Indicating Instruments. LESLIE F. CURTIS.

The Dielectric Constant of Silk. F. J. ROGERS.

The Mathematical Structure of X-Ray Spectra. R. T. BIRGE.

The Spectroscopic Committee of the Division of Physical Sciences of the National Research Council. C. E. ST. JOHN.

Astronomers, mathematicians and physicists participated in a symposium on Einstein's Theory of Relativity June 17, at 2 o'clock, in Denny Hall. Dr. Eric T. Bell read a paper on the principle of general relativity, and there was a discussion led by Dr. J. H. Moore and Professor E. P. Lewis.

The Spectra of Compound Gases Flowing in Vacuum Tubes. W. H. BAIR.

On Friday evening President Suzzalo and the Committee on Arrangements gave a dinner to members of the affiliated societies.

E. P. LEWIS,
Local Secretary for the Pacific Coast.

A Catenary Loaded at One Point.

By S. H. Anderson.

A FLEXIBLE string, hung between two points at *different* horizontal levels and loaded at some point between the two supporting points, assumes a conformation with a point of discontinuity at the position of the load, which divides the string into two arcs. It may be shown that each of these arcs is a portion of a catenary.

From a consideration of the forces acting upon *one* of these arcs a differential equation may be obtained the solution of which gives the two following equations:

$$(y + y_0) = \frac{y_0}{2}(e^{x/c} + e^{-(x/c)}) + \frac{s_0}{2}(e^{x/c} - e^{-(x/c)}), \tag{1}$$

$$(s - s) = \frac{y_0}{2}(e^{x/c} - e^{-(x/c)}) + \frac{s_0}{2}(e^{x/c} + e^{-(x/c)}), \tag{2}$$

in which x, y are the rectangular coördinates of any point, P, s is the length of the arc between the loaded point and P, and y_0, s_0, and c are constants or parameters of the curve which depend for their values upon the weight of the string and the load, and are connected by the following equation

$$y_0 = \sqrt{s_0^2 + c^2}. \tag{3}$$

The interpretation of the three constants is as follows: If the arc is extended to a point where the tangent becomes horizontal, s_0 is the length of this auxiliary arc from the point of loading to the point where the tangent is horizontal; c is the vertical distance from this latter point to a horizontal line which may be called the directrix; and y_0 is the vertical distance from the point of loading to the directrix.

If in the above equations s_0 is put equal to zero, which will be the case when there is no load upon the string, (1) and (2) reduce to

$$(y + c) = \frac{c}{2}(e^{x/c} + e^{-(x/c)}), \tag{5}$$

$$s = \frac{c}{2}(e^{x/c} - e^{-(x/c)}), \tag{6}$$

which are the equations for the common catenary. This shows that the loaded catenary is a more general case of which the common catenary may be considered a special case.

The total tension at any point is given by

$$T = w(y + y_0). \tag{7}$$

The vertical component and the horizontal component are given by the following equations respectively:

$$Tv = w(d + s_0), \tag{8}$$

$$Tg = wc, \tag{9}$$

in which w is the weight of unit length of the string.

The equations for the other arc of the string are of the same form, but the parameters y_0 and s_0 are different, and c the same. Hence in general the form of the curve will be different. Let us call these parameters y_0' and s_0'. Then it may be shown that the load is given by

$$W = w(s_0 + s_0'). \tag{10}$$

This ideal case may be considered an approximation to the problem of the aërial tramway as is used in the logging "skyline" for transporting logs out of the forest. The chief factor of the practical problem that is neglected in the above treatment is the stiffness of the cable. However the length is generelly very great as compared with the diameter so that the relative stiffness is very slight except at the ends where supported and at the carriage where the load is applied.

Perhaps the most valuable feature of the above analysis is the determination of the factors necessary to compute the load a given cable will carry. They are as follows: (1) the horizontal distance between the end points of the cable; (2) the vertical distance between these points; (3) the length of the cable, or the position (with reference to one of the end points) of the lowest point in the sag of the cable that the topography of the ground will allow; and (4) the breaking strength of the cable.

UNIVERSITY OF WASHINGTON.

GIBBS THERMODYNAMICAL MODELS.

BY W. P. BOYNTON.

GIBBS first suggested the representation of the thermodynamical properties of a substance by a three-dimensional model having for coördinates the volume, entropy and energy. Maxwell constructed such a model, and in his "Theory of Heat" presented a diagram of the principal lines upon it. The writer has also constructed a model intended to represent the model described by Maxwell. Finding difficulty in coördinating its form and the location of the lines upon it with the known properties of substances, he computed these lines for a substance following van der Walls' equation, with the ratio of its specific heats 1 2/7, and constructed a model from these graphs. Lately he has recomputed for a van der Waals' substance having the ratio of the specific heats 1 2/3, the maximum value, and constructed a model to correspond, and has also constructed a model, modified to show the properties of the solid state also, which is not included in the van der Waals' equation. These models must be at least qualitatively correct, and show in a general way what the surface must be like for an actual substance.

The moulds for the models have been preserved so that more copies may readily be made. The writer has also drawings and lantern slides showing contours and projections of principal lines upon the planes, and the appearance of the finished models.

UNIVERSITY OF OREGON,
EUGENE, OREGON,
May, 1920.

Characteristics of Vacuum Tubes.

By S. L. Brown and C. E. Normand.

SEVERAL types of vacuum tubes for detection, amplication and generation of radio-frequency currents have been examined in order to compare types of tubes and different tubes of the same type. The different types of vacuum tubes examined include those from American, French and German manufacturers.

Data Showing Characteristics of Vacuum Tubes at their Rated Filament Current and Plate Voltage.

Type I: Western Electric, VT 1; filament current 1.1 amp., plate voltage 20 volts.
Type II: General Electric, VT 11; filament current 1.1 amp., plate voltage 30 volts.
Type III: General Electric, VT 14; filament current 1.4 amp., plate voltage 400 volts.
Type IV: Marconi, Class I; filament current .6 amp., plate voltage 40 volts.
Type V: French, Bis 3; filament current .6 amp., plate voltage 20 volts.
Type VI: German, EVN 117; filament current .6 amp., plate voltage 50 volts.

Type.	Grid Voltage.	Plate Current.	Type.	Grid Voltage.	Plate Current.
I	−10 v.	00 ma.	IV	−20 v.	00 ma.
"	00	.75	"	−10	.05
"	10	2.65	"	00	.50
"	20	10.10	"	10	2.05
"	30	12.00	"	20	2.15
"	40	14.50	"	30	2.05
"	50	15.00	"	40	2.00
II	−10	00	V	−20	00
"	00	1.00	"	−10	00
"	10	2.05	"	00	.10
"	20	4.10	"	10	1.50
"	30	6.00	"	20	3.55
"	40	7.25	"	30	3.80
"	50	7.15	"	40	3.80
III	−50	00	"	50	3.70
"	−40	.45	"	60	3.65
"	−30	1.10	"	70	3.65
"	−20	3.25	VI	−20	00
"	−10	10.10	"	−10	.10
"	00	12.55	"	00	.50
"	10	12.80	"	10	1.00
"	20	13.00	"	20	2.10
"	30	13.10	"	30	3.55
"	40	13.15	"	40	4.00
"	50	13.20	"	50	4.10
"	60	13.20	"	60	3.80
"	70	13.20	"	70	3.40

University of Texas.

TRANSPARENCY TO TOTAL HEAT RADIATION.

BY S. L. BROWN.

THIS paper gives the results of measuring the percentage of total radiation transmitted by glass, rock-salt, mica, celluloid and paraffin for temperatures of radiating source from 200° C. to 1200° C. This work was undertaken to determine the possibility of selecting pyrometer screens which would intercept a known fractional part of the total radiation at a particular temperature. Results show that the per cent. of total radiation transmitted by ordinary sodium glass increases from about 6 per cent. for 200° C. as the temperature of the source to about 60 per cent. when radiating source is at 1200° C. The transparency of mica increases from 7 per cent. to 40 per cent. when the temperature of the radiating source is increased from 400° C. to 1200° C.

UNIVERSITY OF TEXAS.

A STUDY OF THE RESIDUAL IONIZATION IN A GAS WITH REFERENCE TO TEMPERATURE EFFECTS.

BY C. H. KUNSMAN.

INVESTIGATIONS were undertaken with the purpose of studying the ionization of a gas as observed in an airtight vessel. This was done with a twofold purpose: (1) to find an accurate method of measuring this conductivity in the gas, (2) to make a thorough investigation of the effect of temperature on this conductivity. A double ionization chamber one compartment containing the electroscope, made it possible to eliminate insulation losses by taking observation with the compartments connected or with the electroscope chamber above.

Tests were undertaken at low temperatures for two reasons: (1) if ionization is produced by molecular impact due to thermal agitation, such an effect could best be investigated by drecreasing this molecular motion as much as possible, (2) there is no record of such tests being made at low temperatures. The electrometer was rigidly clamped in a frame and a Dewar flask containing solid carbon dioxide and alcohol raised until the ionization chamber was completely surrounded wish the cooling solution. During this change of temperature of the gas within the chamber, very pronounced variations of the position of the leaf was observed. For some time, depending upon how rapidly the temperature was lowered, the rate appeared to be normal. With a further decrease in temperature the rate of loss of charge became quite rapid, and with a further lowering of temperature the charge on the insulation system disappeared entirely. The ionization chamber was then disconnected from the charged system and a similar set of results obtained. This at once shows us that the increased conductivity takes place across the insulation and is not due to a variation in the ionization in the enclosed gas.

Separate tests on various insulations including sulphur, indicate that the increased conductivity is due to traces of water vapor which have no notice-

able effect except at low temperatures. Sulphur proved to be the best insulator under all conditions and through a wide range of temperature.

For investigations above room-temperature, the ionization chamber was placed in an electric heater. Both the sulphur insulation and the wax joints were kept cool by water jackets. No increase in conductivity was noticed until the temperature of the enclosed gas was between 95° C. and 100° C., when a pronounced increase was noticed. This increase was again noticed when the ionization chamber was disconnected from the charged system at about the same temperature, which proves that the increased conductivity takes place across the insulation support and is not due to an increased conductivity in the gas.

From these experiments we may conclude:

1. The mean value of the electrical conductivity of air in the basement of the physical laboratory at the University of California was 8.22 ions per c.c. per second.

2. The mean value of the electrical conductivity as measured on the Pacific Ocean was 4.15 ions per c.c. per second.

3. This electrical conductivity, or residual ionization, is not due to molecular impact of thermal agitation.

4. The apparent increase in ionization as observed at high altitudes is solely a temperature effect, and is due to an increase in conduction over the insulation.

5. The effect of change in temperature on the insulation system of an apparatus of this kind is sufficient to account for the apparent daily and seasonal variations of the residual ionization as reported by some observers, and is not due to a variation of the ionization of the gas within the chamber.

University of California,
May, 1920.

The Continuous Spectrum of Hydrogen in the Schumann Region.

By E. P. Lewis.

REFERENCES have frequently been made to a continuous spectrum of hydrogen associated with the spark discharge at high pressure or with vacuum tube discharges at low pressure, but the conditions most favorable to its appearance and its extent have not been discussed. Kayser rather takes it for grantedt hat an intense discharge through a small capillary is the most favorable condition, and Schniederjost used that method, with poor success, for giving a background for absorption spectra. The writer pointed out some years ago that this spectrum could be best produced by using a rather large end-on tube, say one cm. in diameter, with a simple discharge of small current density, at a pressure of five mm. or more, and that it was greatly weakened by the use of a condenser and spark gap.[1] This spectrum extended to the limit of transmission of the quartz system, its gradual weakening apparently being

[1] Science, 41, 947, 1915.

determined rather by the increasing absorption of the quartz and the diminished sensitiveness of the photographic film than by failure of emission. There was no sign of resolution into lines, and it extended well into the visible region as well to in the opposite direction, although the superposition of the many-lined spectrum made it impossible to determine its exact limit. The writer has recently had constructed a fluorite vacuum spetrograph, which made it possible to extend observations into the Schumann region. As anticipated, it was found that the continuous spectrum extends with undiminished intensity well into this region. At a wave-length of about 1800 it begins rather suddenly to diminish in intensity, and disappears about 1750. Faint bands attributed by Schumann and by Lyman to carbon monoxide make observations in this region somewhat uncertain. A little below 1700 the line spectrum of hydrogen reappears, with no trace of a continuous background. Lyman refers to an absorption band of unknown origin in this region, but the fact that the continuous spectrum does not reappear with the lines shows that the limit is not far from the point specified. There may be some theoretical interest attached to the fact that this continuous spectrum completely fills the region between the Balmer series and the Schumann line spectrum, with no lines whatever superimposed on it.

This continuous spectrum offers the best background yet obtained for the study of absorption spectra in the extreme ultra-violet. Its perfect uniformity and continuity makes it far superior to the aluminum spark under water. Good photographs of the absorption spectrum of benzol have been obtained with an exposure of fifteen minutes. The absorption appears to be complete beyond about 1900.

Lyman makes a casual reference to a continuous spectrum, and Schumann used it in the study of absorption spectra, but neither appears to have investigated the most favorable conditions for its appearance.

Helium and neon resemble hydrogen in giving continuous spectra far into the ultra-violet, which are not, however, so intense as that of hydrogen. The continuous spectra given by parts of some nebulæ may be due to hydrogen and helium.

UNIVERSITY OF CALIFORNIA,
May, 1920.

MULTIPLE REFLECTIONS FROM THE INTERIOR OF A RING.

BY A. A. KNOWLTON AND G. A. WATT.

LIGHT from a point source reflected at a concave cylindrical mirror of large aperture gives rise to the well known caustic. If the reflecting surface forms a complete ring multiple reflections occur which produce caustics of higher orders. Slides prepared from photographs showing these curves for the second and third order were shown. The equation for the nth caustic has been obtained by a simpler method than that used by Cayley, who treated the theory of such reflections.

REED COLLEGE,
May, 1920.

An Optical Illusion.

By A. A. Knowlton and G. A. Watt.

If one stands with his back toward a window and looks into an ordinary flask nearly filled with clear water and held slightly above the level of his eyes he appears to see, near the bottom of the flask a bubble of oil or other highly refractive substance. This is in fact an image of the plane surface of the water distorted by reflection in the spherical mirror of large aperture and by refraction. Examples of this phenomena were shown and the cause of the illusion discussed.

Reed College,
May, 1920.

Velocity of Sound from a Moving Source.

By R. B. Abbott and J. W. Cook.

OBSERVATIONS have been made which do not show any change in the velocity of sound due to a spark source moving *8550 cm. per second.* The source consisted of a spark gap mounted on the end of a vane rotating 30 R.P.S. The spark was made to occur at the same point once in a revolution. Opposite points on the spherical wave front were located by two telephone transmitters and one receiver connected to a differential transformer. A minimum sound in the receiver indicated that the wave front arrived at the two transmitters simultaneously. The radii of the spherical waves measured varied from 190 to 260 centimeters. Normal room temperature and pressure prevailed.

The conclusions are that the velocities of sound disturbances produced by a moving spark are not appreciable affected by the velocity of the gap.

Whatever increment of velocity may have been given to the sound by the motion of the gap, was damped out too quickly to affect the measurements beyond the errors involved.

University of California,
May 25, 1920.

Voltage Wave Analysis with Indicating Instruments.

By Leslie F. Curtis.

AN investigation and instrument are described which were inspired by the lack of a suitable workable standard of wave form. The use of telephone condensers, air core inductances, and a sensitive hot-wire galvanometer to determine the order and amplitude of the harmonic components of a wave—leaving only their phase positions undetermined—is described. The method of analysis is based on the well-known use of the wave meter, but adapted to low frequencies and with this important difference—a fundamental frequency is always present, may not be neglected, and must be treated simultaneously

with the other harmonics. The equations used are developed and explained under the headings "Order" and "Amplitude" of the Harmonics. The sensitiveness of indication is discussed and applied to several types of circuits. A convenient arrangement of the constants of the circuit in an instrument called by the author an "Harmonimeter" and giving direct readings of the order of the harmonics and easily calculated values of their amplitude is illustrated. An example of wave form measurement is given in which the results are checked by oscillogram.

UNIVERSITY OF WASHINGTON,
SEATTLE, WASH.

THE DIELECTRIC CONSTANT OF SILK.

BY F. J. ROGERS.

SILK does not appear in any of the usual tables of dielectric constants, probably no such measurements have ever been made.

In general, when two dielectrics are electrified by contact and friction, the one having the larger constant becomes positive. If this holds true with silk and other substances then its dielectric constant must be rather large. This is what suggested the measurement of this constant for silk.

The usual condenser method was adopted. As a silk fabric is not a continuous solid, some method must be used which will allow for the space between the condenser plates which is not filled with silk. This was done in two ways. First, several thicknesses of silk were used and the thickness measured while weighted between the condenser plates. The ratio of silk to air was then computed from the weight of the silk, the total volume, and the density of the silk. The density of the silk was determined by the simple expedient of weighing it in air and in water. The density proved to be surprisingly large, but as the silk fibers are solid and of a horny nature this ought to have been expected. In the second method the silk was dipped in melted paraffin and the "paraffined" silk used between the condenser plates. In this case to insure contact the lower plate was amalgamated zinc with a large excess of mercury and the upper plate was liquid mercury. In both cases it was assumed that the dielectric contact of the mixed dielectric was a simple additive function of the two dielectric constants for silk and air and for silk and paraffin.

The simple bridge method was used for measuring the capacity of the condenser having silk as a dielectric. It formed part of one of the condenser arms of the bridge and its capacity was determined by the substitution of variable calibrated air condenser. Alternating E.M.F. of 60 cycles was supplied to the bridge, and a quadrant electrometer was used as an indicating instrument. The electrometer needle was maintained at a high alternating E.M.F. of the same frequency and in quadrature with that supplied to the bridge.

The experimental results obtained were as follows:

	Specimen.		
	White Satin.	Black Satin.	White Silk.
Density	1.51	1.515	
Dielectric constant	4.62	5.07	4.62
Silk and air exp.	4.68	5.07	4.68
Dielectric constant	4.58	4.86	4.58
Silk and paraffin exp.			

STANFORD UNIVERSITY,
May, 1920.

THE MATHEMATICAL STRUCTURE OF X-RAY SPECTRA.

BY R. T. BIRGE.

THE mathematical relations of X-ray spectra may be divided into two classes, (1) those concerning the same spectral line as it appears in the spectra of successive elements (Moseley's $\sqrt{\nu} : N$ curves) and (2) those concerning the various lines of any one element. Using the very accurate data now available, the author has tested many of the previously proposed relations, as well as some new ones. The work is still in progress.

Under (1), the Siegbahn 1919 data for nine $K\alpha_2$ lines, extending from $N = 17$ to $N = 29$, plus that for N = 74, have been fitted to an ordinary power series equation. A fourth degree equation (five undetermined constants) was found entirely satisfactory, the average *O-C* values being 1/300 per cent., with no evidence of systematic divergence. The agreement for the smaller atomic numbers is especially good, and so the extrapolation to hydrogen and helium can be made with some confidence. The value for $N = 1$ is 2535 Å., with a possible error of not more than a few hundred Å. (Uhler obtained 400 Å. or less.) The theoretical "resonance" line is 1216 Å. For helium the extrapolated value is 470.7 Å.,—the resonance line 567 Å. Theoretical considerations indicate that H and He have no $K\alpha_1$ line, and so the disagreement is to be expected. The author's curve has an intercept $N = +0.23$.

The general equation of the second degree (a special form of which was used by Uhler) was found unsatisfactory. Duane has proposed a linear relation between velocity and atomic number, for *K* critical absorption, but his formula for kinetic energy, from which the velocity was obtained, is incorrect, and therefore the results must be considered purely empirical.

It is noteworthy that the $\sqrt{\nu} : N$ curves are smooth, even for the lowest atomic numbers, and that each curve seems to begin when there are only two electrons in the ring (or shell) from which the emitting electron comes. *None* of the graphs are linear, although in some cases the slope varies only about 10 per cent.,—in others 40 per cent. to 50 per cent.

The only satisfactory relations, under the second type of regularity, are given by Sommerfeld in a chart.[1] Even these are only partially satisfactory

[1] Zeitschrift für Physik, Vol. 1, page 137, 1920.

since $K\alpha_1 + L\alpha_1 + M\alpha_1 + N\alpha_1 +$ etc. does *not* equal K absorption. In fact, it would be necessary to change all Stenström's M series data (both emission and absorption) by about 15 per cent., in order to obtain a general agreement. The combination principle appears strictly accurate in certain restricted types of relations, but quite inaccurate in general.

The fact that certain $\sqrt{\nu} : N$ curves of the L series *cross* one another proves that there must be various *independent* configurations of the atom. Thus far, in the theoretical X-ray work, only *one* set of related orbits has been considered. For ordinary spectral series there are *four* such sets of orbits (fundamental, principal, sharp, and diffuse.)

D. L. Webster's relations between emission lines and critical absorption are extremely significant, but the fact that the L_2 critical absorption curve[1] *crosses* the γ_2 emission curve, raises a vital objection, on quantum theory, to the association of γ_2 with L_2 absorption. There are other similar cases.

Overn's eight equal frequency ratios for Tungsten are interesting, but unfortunately only two (β_1/α_1 and γ_1/β_3) can be tested for other elements. The two ratios are *not* equal except for Tungsten, the curves (ratio against N) crossing at $N = 74$. Can it be possible that all eight curves will be found to cross at the *same* point? Additional experimental data is much to be desired.

UNIVERSITY OF CALIFORNIA.

THE SPECTROSCOPIC COMMITTEE OF THE DIVISION OF PHYSICAL SCIENCES OF THE NATIONAL RESEARCH COUNCIL.

BY CHARLES E. ST. JOHN.

IT is not necessary to recall to this body the part played by the Nationals Research Council in organizing the scientific resources of the country for the purpose of winning the great war. It was, however, a revelation to the country as a whole of the strength and weakness of the pre-war conditions in scientific work. The great success with which men trained in pure science attacked the various practical problems of the war increased the estimation in which scientific research was held and has made it possible to carry on under peace conditions some of the benefits of organization and to increase the fostering agencies of research in pure science. In line with such considerations and the executive orders issued by the President outlining the purposes of the Council, the National Academy reorganized and put upon a permanent basis the National Research Council whose purpose is "to promote research in mathematical, physical, and biological sciences, and in the application of these sciences to engineering, agriculture, medicine, and other useful arts, with the object of increasing knowledge, of strengthening the national defense and of contributing in other ways to the public welfare." In paragraph 3 of the order issued by the President an important point is expressed as follows: "to promote coöperation in research, at home and abroad, in order to secure concentration of effort, minimize duplication, and stimulate progress; but in all

[1] Duane's 1920 data, not yet published.

coöperative undertakings to give encouragement to individual initiative as fundamental to the advancement of science."

In furtherance of the purposes of the Council the Division of Physical Sciences, among others, has been organized of which Dr. C. E. Mendenhall is chairman. Other members of the Division are representatives of the related societies. In accordance with the provisions of the Council, the Division of Physical Sciences has recently established a General Spectroscopic Committee of ten members divided into an Eastern and a Western group. Before entering into the committee's plans in detail it may be of interest to refer to the financial situation.

The Rockerfeller Foundation has entrusted the Council with the expenditure of $500,000 within a period of five years for providing fundamental research in physics and chemistry, primarily in educational institutions of the United States. This purpose is to be carried out through establishing a system of fellowships to be awarded to American citizens whose training is equivalent to that represented by the doctor's degree.

Important results are hoped from this undertaking and such a degree of success in accomplishing the ends in view that conditions in American centers of research will be permanently improved. Some of the results expected are:

1. Opening of a scientific career to a larger number of investigators and their more thorough training in research, thus meeting an urgent need of our Universities and industries.

2. Increase of knowledge relating to the fundamental principles of physics and chemistry.

3. Creation of more favorable conditions for research in this country.

In addition to the fellowship funds placed at the disposal of the Division of Physical Sciences, the Rockerfeller Foundation has also appropriated funds to meet the expenses involved in conferences of special committees and in the honoraria of investigators to whom the preparation of special reports, surveys, or monographs is committed. In fact the funds necessary to set the machinery in action are provided and it now remains to be seen how men of science, young and old, rise to the opportunities. It may well be that the next few, years will be crucial ones in developing physical science in the United States and it behooves us all to aid in all ways we are able, either by joining in or promoting interests in research, by preparing others for it, or by helping to create an understanding and hence a sympathetic attitude on the part of the general public toward increase in our knowledge of the fundamental facts and principles of physics, irrespective of their immediate practical applications.

The Eastern group of the Spectroscopic Committee consists at present of Lyman, Chairman, Gale, Randall, Saunders, and Uhler, the Western Group of Anderson, Babcock, King, Lewis, and St. John.

The activities of the committees are as yet but partly organized, but they hope so serve as clearinghouses of information as the means of correlating the work so that needless duplication may be avoided, and of bringing into closer relations men who are working along similar lines in different parts of the

country. It is already evident that men are going to appreciate the contacts with the outside which the committees supply, and that the committees will be able to render assistance in approaching the administrators especially of our state universities in the interests of fundamental research, helping to create a more favorable atmosphere and to bring about a more generous provision for research.

At present a canvass of the universities is under way to ascertain the equipment and opportunities available and the lines of investigation in progress. Such a body of data will be of great service to the committee when responding to requests for suggestions as to lines of investigation that will fit into general plans or that need to be undertaken.

It is expected that subcommittees, membership not confined to the main committee, will soon be organized for discussion and report on special fields which are best suited for or are in need of such a survey. The consideration in view would be as follows:

1. They would be stimulating and valuable to the men themselves.
2. The groups might undertake to interest others in their special fields.
3. The reports would be of value to all physicists in helping them to get into touch with various phases of spectroscopic work.

Correlated with the work of the general committee is that of the International Commission for Standards of wave-length. The American Committee reporting at the Washington meeting preliminary to the Brussells conference was unanimous in the opinion that one of the greatest needs of the physicist at the present time is a complete determination of the wave-lengths of the elements with the utmost accuracy possible. Such an undertaking will require the united work of many investigators but its value will be commensurate to the cost in time and effort. It is well suited to coöperative efforts, as each investigator or small group of investigators can have a definite field and feel the satisfaction that he is contributing an element of value to the larger work. Experience has shown that such investigations, besides accomplishing the important ends directly in view, can hardly fail to yield valuable results in related fields, in particular in connection with problems of radiation, and in the relations between spectra of terrestrial and cosmic sources.

There is much said of correlation and coöperation but it must ever be borne in mind that the greatest asset in research is individual initiative and that in all our efforts at coöperation the aim should be to discover and recognize early the fortunate possessor of this rare and prescious gift and the utmost care be taken that in our desire for efficiency of organization the machinery does not hinder its free and full development.

MT. WILSON OBSERVATORY.